Indoor Localisation In Wi-Fi-Networks Using An Improved Centroid Approach

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Abbreviations

AOA	Angle of arrival
AP	Access point (Base Station)
BER	Bit error rate
BS	Base Station (Access point)
CP	Centroid point
dB	decibels
dBm	power ratio in decibels of the measured power referenced to one milliwatt
e.g.	exempli gratia – for example
FAF	Floor attenuation factor
f.	and following page
ff.	and following pages
i.e.	id est - that is
LOS	line of sight
MS	Mobile Station
mW	milliwatt
NLOS	non line of sight
p.	page
RF	Radio frequency
RSS	Received signal strength
RSSI	Received signal strength indication
TDOA	Time difference of arrival
TOA	Time of arrival
WAF	Wall attenuation factor

Abstract

This thesis studies triangulation with Wi-Fi networks inside buildings. The centroid algorithm is adapted to account for complex environments by moving the centroid according to the received signal strength (RSS) from a mobile station (MS) to at least three base stations (BS) weighted with RSS between BSs (inter BS weighting). This helps to account for sources of attenuation (e.g. walls, furniture, people). A discussion of situations on the triangle helps in developing an inside/outside test for the situations, where the MS is not located inside the triangle formed by the BSs (outside-cases). The developed model is tested with available data and the performance of different algorithms is discussed. Since the data available is too sparse to efficiently estimate the quality of the approach for narrow time bands, further studies are encouraged. The results show that the accuracy of estimation is improved by applying a weighting factor to the received signal strength readings between mobile station and base station. This weighting factor is calculated by comparing the received signal strength between the base stations to the expected signal strength on the same distance. The identification and the handling of outside-cases turned out to be insufficient to improve the weighted centroid algorithm.

1 Introduction

Knowing the location of a desired person or thing solves multiple problems in a lot of different situations. In situations, where a position needs to be tracked outdoors, GPS is often the technology to look for. However sometimes, GPS might be affected in accuracy or usability. This is mainly the case, when GPS is shadowed (clouds, trees, buildings) or when the recipient is indoors (Huang et al. 2011, p. 325).

Situations where indoor positioning is needed include indoor guidance in unknown buildings (e.g. airports) and structures. A non people centric application is keeping track of objects. Kolodziej et al (2006, p. 3ff.) lists multiple use cases and possible goals. Depending on the technology used, different possibilities and challenges arise[1].

Due to the rising distribution of cell phones, suitable technologies normally used for communication (including GSM, Wi-Fi, Bluetooth, ultrasound, see appendix I) are subject to tracking efforts. (Kolodziej et al. 2006, p. 226f, Liu et al. 2007, p. 1077). The most promising approaches to tracking use technologies that do not rely on the cell phone users to install software. Thus a barrier is eliminated, when the user is voluntarily[2] or involuntarily[3] subject to tracking efforts.

1.1 Problem Formulation

Multiple approaches for indoor and outdoor localisation are possible (Liu et al. 2007, p. 1068ff). These include:

- Proximity - nearest base station
- Calculations with received signal strength indication (RSSI) - triangulation
- Time difference of arrival (TDOA) - triangulation
- Fingerprinting - site survey

All these approaches come with their own challenges and limitations, mostly the lack of information or the availability of only imprecise information, the need to constantly update available information due to a changing environment or overall bad performance.

Approaches with the best performance include the ones from the fingerprinting family. However fingerprinting techniques are only feasible if a structure is constantly mapped (Koweerawong et al. 2013, p.414). In structures not yet mapped for localisation (ad hoc) or with outdated information, localisation approaches based on fingerprinting lack the desired

[1] An overview will be given in Chapter 2
[2] e.g. to find nearest printers in an unknown building, where the building provides a localisation service
[3] e.g. for marketing or security reasons

precision. While fingerprinting approaches are more precise than the ones of the geometric family (proximity and triangulation), the latter ones are easier to use.

Changing environments are an important factor in indoor localisation situations. The fingerprinting and the geometric approaches are negatively influenced in precision by changing factors (moving people, objects) that lead to a change in signal distortion. Estimating and accounting for the dynamic changes in signal distortion would therefore lead to greater accuracy.

1.2 Focus, Goals and Overview

The goal of this bachelor thesis is to study indoor tracking capable of adapting to dynamic changes. A further goal is to investigate a novel idea to improve the localization accuracy of geometric approaches, thus keeping their ease in application (no decaying fingerprints, simple geometry) while getting closer to the performance of fingerprinting approaches.

To reach these goals, algorithms able to cope with dynamic changes of the situation will be developed and tested with real world data provided by the university ("Location Based Analyser" Eurostars project E!5533). The approaches make use of information gained from the wireless network and can therefore not be of the fingerprinting family. This information is the RSS received from the mobile station (MS) and the RSS between the base stations (BS). No other network information or sensory type needs to be used, no software needs to be installed on the mobile station (MS) in question. The algorithms will be tested and compared with available data by means of a model developed in Matlab.

1.3 Outline

As this first chapter provided an introduction to the subject of the thesis, Chapter two will provide further necessary information to understand the terms and techniques used. As the literature on the field of wireless tracking is huge, an accurate survey of the field in question is not the scope of this thesis. Still, different possible approaches will be presented and discussed. In Chapter three an improved version of the weighted centroid algorithm will be developed by means of analytical discussion. In Chapter 4, the triangulation model developed and the algorithms will be presented. Real world data will be introduced and analysed. The results gained from running the algorithms will be discussed. Chapter 5 will contain a summary and outlook for further studies.

2 Background

This chapter lays the theoretical base with regards to the subject. Therefore essential terms and concepts are introduced including propagation theory and path loss models. To calculate a location, different measures are needed. These include the received signal strength indication (RSSI). Therefore in Chapter 2.2., the author shows how to calculate a distance reading out of RSSI. In Chapter 2.3. localisation methods are presented.

2.1 Devices

An **access point** or **base station** (BS) is a router distributing a wireless signal. In most deployment scenarios these have fixed positions. Furthermore, the positions are known and can be used by the localisation algorithms.

A **mobile station** (MS) is a mobile wireless receiver (smartphone, tablet, laptop, wireless tag, anything that sends a wireless signal that can be tracked). The goal of localisation is to estimate the unknown position of the mobile device as accurately as possible.

2.2 Between sender and receiver

In wireless networks (examples are WLAN (IEEE 802.11) or cellular Networks (GSM)) information is transmitted over electromagnetic waves. It is therefore not possible to study localisation without basic understanding regarding the subject.

2.2.1 Power density of an electromagnetic wave

The power density of an electromagnetic wave is proportional to the transmitted power and inversely proportional to the square of the distance to the source (Bensky 2008, p. 2).

The following formula shows Friis' equation on free space-propagation (Bensky 2008, p. 140):

$$P_r = P_t * G_t * G_r * \left(\frac{\lambda}{4\pi d}\right)^2 [mW]$$

where P_r is the received power, P_t the transmission power of the sender, G_t the transmitter antenna gains, G_r the receiver antenna gains, λ the wavelength and d the distance between sender and receiver.

Obviously, the formula is not valid for a distance of 0 meters. Many propagation models therefore use a different representation for near distance. These locations close to the transmitter are called near distance reference points, typically chosen to be at 1m. (Sarkar 2003, p. 52)

In most cases (e.g. real world scenarios not taking place in space) the Friis' equation might not be a sufficient accurate model of reality. Extensions and changes to the basic equation are a necessity.

2.2.2 Calculating attenuation

Path loss

Path loss is defined as „[...]the attenuation undergone by an electromagnetic wave in transit between a transmitter and a receiver” in a communication system (ATIS 2000). The formula of path loss is given by

$$Path\ loss = L = 10 * log\left(\frac{P_r}{P_t}\right)\ [dB]$$

with P_r being the power at the receiver and P_t the power transmitted. If the transmitted power is bigger than the received power, the resulting value is negative and denotes a path loss, with a bigger received power amplification ensues.

RSSI

The abbreviation RSSI stands for received signal strength indication. In embedded devices, the received signal strength is converted to RSSI which is defined as the ratio of the received power to the reference power (P_{Ref}). The formula for RSSI is similar to the path loss formula where the power transmitted by the sender is replaced with a reference power (Sarkar et al. 2003, p. 52)

$$RSSI = 10 * log\left(\frac{P_r}{P_{Ref}}\right)\ [dBm]$$

where P_r stands for the remaining power of the wave at the receiver (Blumenthal et al. 2007, p 2). Typical values of RSSI range from -100 dBm (for a very low signal level) to -60dBm (very strong signal level). (Sauter 2010, p. 160).

Typically, the reference power represents an absolute value of P_{Ref} = 1 milliwatt (mW) (Blumenthal et al. 2007, p 2). Therefore, RSSI is usually expressed in dBm. dBm is the abbreviation for the power ratio in decibels (dB) of the measured power referenced to 1 mW (Sauter 2010, p160).

It is to be noted that there are significant differences between Wi-Fi devices. Devices from the same vendor, even devices of the same model might not perform identically. Furthermore some devices are not able to report valid or useful RSSI or have unusual temporal patterns leading to bigger challenges (Lui et al. 2011, p. 57).

RSSI versus RSS

While RSS is the actual reading in dB, dBm or the like depending on the chosen units, RSSI is an index that is not precisely defined (and could therefore be chosen at will). Converting to RSSI can however aid in interpreting the RSS values. It is to be noted that RSS and RSSI are often not distinguished precisely in papers and books and one is used as synonym of the other.

2.2.3 Sources of path loss

The Friis' equation accounts for factors changing the received power of a signal in free space. There are however no variables accounting for the fact that, under different circumstances, RF-signals are rarely a perfect sphere (as free space propagation presumes) and have to pass different obstacles on their way to the receiver. "For instance, moving the position of one chair, or opening/closing a door can change both multipath fading and slow fading losses" (Abbas et al. 2012, p. 2).

Kolodziej et al. (2006, p. 150) list four environmental factors causing effect on accuracy:

- **Attenuation**: signal strength is changed, as the signal passes a person or object
- **Occlusion**: the signal is blocked completely
- **Reflection**: the signal is reflected off objects (walls, screens, ground) – its path to a sensor is longer, therefore the received signal strength (RSS) will be lower
- **Multipath**: the signal can follow multiple paths before reaching the mobile station – an indirect path lets the BS appear further away. Multipath consists of shadowing (slow fading) and fast fading

Parameswaran et al. (2009, p. 5) name *interference from other objects* and *attenuation caused due to barriers* as main reasons for inconsistent results of RSS-calculations (apart from obvious reasons like power failures or malfunctions). Zanca et al. (2008, p. 4) and Abbas (2012, p. 4) add moving people to the factors that need to be considered. Sarkar et al. (2003, p. 54f) names *diffraction* and *scattering* as further propagation mechanisms, which affect the signal. Furthermore according to Yang et al. (2010) the layout of the nodes that are used in triangulation play an important role as some node layouts lead to better results.

Considering attenuation, humans play an important role. Since WLAN signals are mostly transmitted over 2.4 GHz, what is also the resonance frequency of water, humans, consisting to roughly 3/4 of water significantly absorb WLAN signals. At 1 meter distance between MS and BS, the difference between facing and looking away from the BS while holding the MS results in a loss of over 40 dBm. Stronger signals tend to show greater attenuation than weaker ones, therefore at 10 meters, the attenuation is only around 5 dBm. (Fet et al. 2013, 499ff)

Adjustments of the Friis equation to account for different factors depending on the surrounding environment are therefore needed. This is done using path loss models. The following chapters show different path loss models for outdoor and indoor propagation.

2.2.4 Outdoor propagation

Urban Okumura Hata Model

The urban Okumura Hata model is an empirical formulation of the path loss data from Okumura's original model. It is the most widely used propagation model for the behaviour of cellular transmissions in areas with buildings. As in the original Okumura model, the median path loss is calculated. Formulas differ between settings, e.g. small- to medium-sized cities, large cities, suburban or rural areas. The model is suitable for mobile systems that cover cells of more than 1 km in radius. The formula is given by

$$L_{50} = 69.55 + 26.16\log(f) - 13.82\log(h_B) - a(h_M) + (44.9 - 6.55\log(h_B))\log(d)\ [dB]$$

where *f* denotes the frequency [MHz], h_M the height of the MS and h_B the height of the BS [m], *d* the distance between MS and BS [m]. *a()* stands for the correction factor for the effective antenna height, in the following formula shown for the small-to medium-sized city-case[4] (Sarkar et al. 2003, p. 56):

$$a(h_M) = (1.1\log(f) - 0.7) * h_M - (1.56\log(f) - 0.8)[dB]$$

Other outdoor models

Different models include different components to calculate path loss, depending on the situation where the model is applied. The following is by no means a complete overview of outdoor models and gives an overview of two further approaches.

As another urban model, the **COST-231-Walfisch-Ikegami model** adds roof-top-to-street diffraction and scattering loss and multi-screen diffraction loss to the free space path loss. The width of the street where the MS is situated, the height distance between MS and BS, the angle of incidence relative to the direction of the street and again an urban density factor are part of the equation.

The **dual-slope model** on the other hand describes a line of sight situation and is based on a two-ray model, where added path loss significantly increases after a critical distance. This

[4] For other cases see Sarkar et al. 2003, p. 56

model needs the height of transmitter and receiver antenna along with the distance between them to calculate the path loss. (Sarkar et al. 2003, p. 56f).

2.2.5 Indoor propagation

As seen in the previous chapters, radio signals are absorbed and reflected by walls, furniture and people. Indoor approaches therefore have to account for different settings and layouts. Different approaches for calculating indoor path loss exist.

Log-Distance path loss model

The log-distance path loss model is a site-general model. Its formula is given by

$$L = L(d_0) + N * log_{10}\left(\frac{d}{d_0}\right) + X_s \; [dB]$$

where $L(d_0)$ is the path loss at the reference distance d_0, d is the distance between BS and MS, N/10 is the path loss distance exponent and Xs is a random Gaussian variable with zero mean and a standard deviation of σ. N and σ depend on the structure and layout of the building and the frequency used[5]. (Seybold 2005, p. 214f)

Sarkar et al. (2003, p. 58) list a similar model, where a floor- and wall-attenuation factor is added, depending on the number of walls or floors the signal has to pass. Here FAF and WAF stand for the floor respectively wall attenuation-factors.

$$L = L(d_0) + N * log_{10}\left(\frac{d}{d_0}\right) + \sum_{q=1}^{Q} FAF(q) + \sum_{p=1}^{P} WAF(p) \; [dB]$$

A further source, where the angle of incidence on the wall or floor is accounted for, is also indicated. However, in both cases the random variable is not part of the equation.

A mixture of both approaches is used by Kim et al. where the random variable reappears. Due to the simpler situation analysed (only one floor), only the wall-attenuation factor is included in this model. (Kim et al. 2011, p. 934)

Markov models

Not only the number of walls but also the number of people has an impact on radio waves and therefore influences the path loss in a building. Depending on time and day, a different number of people stay in a building. A static distribution of the fading factors is therefore an unrealistic assumption. A higher precision can be reached by using a Markov chain for the supposed distribution of the path loss depending on time and day of the week, where the probabilities between the states change according to indoor RF channel conditions,

[5] For typical values see Seybold 2005, p. 214

transceiver positions or the sampling rate (Abbas 2012, p. 6). While this approach might be more precise than the standard log-distance path loss models, it is more complex and asks for a constant monitoring to adjust the distributions used.

2.2.6 Calculating distance from RSSI

In some cases, distances are needed to estimate the location of a mobile station. RSSI as a measurement does not directly help in triangulation, but distances can be derived from RSS-measurements. Rearranging the RSS calculation equation to P_r, with P_{ref} set to 1mW, leads to the following equation.

$$P_r = 10^{(RSSI/10)}\ [mW]$$

The distance can be calculated using Friis's formula or an adaptation to account for the environment (buildings, walls and the like).

An increasing received power results in a rising RSSI, thus, the distance *d* is indirectly proportional to RSSI. Still, even under ideal circumstances and in a controlled environment, real and derived distances differ (Kumar et al. 2009, p. 3). Due to the difficulties to develop a reliable match between RSSI and distance, Parameswaran et al. (2009) go as far as to propose that RSSI should not be used as a metric for distance measurements in localisation.

2.2.7 Alternative indicators

LQI

RSSI is not the only indication used in communication systems. Link quality indication (LQI) is a different signal indication. Depending on the provider, it may be implemented using receiver energy detection (RSSI), a signal-to-noise ratio estimation or a combination of the two. (Bensky 2008, p. 255f) The resulting value is defined to be between 0 and 255. Systematic outliers based on channel effects are observable. It can be used for distance measurement or location as the LQI decreases with an increasing distance. (Blumenthal et al. 2007, p. 2).

Due to the definition of LQI and the allowed possible differences in calculations, the usage of LQI in localisation adds further sources of error, while it suffers from the same problems as RSSI (signal reflection and occlusion).

TOA & TDOA

Time distance of arrival (TDOA) or Time of arrival (TOA) are approaches, where not the signal strength is measured, but the time it takes the signal to reach its destination. TOA measures the time it takes the signal to get from a MS to a BS. With the speed of an electromagnetic wave being the speed of light, the distance can then be calculated and used

in trilateration (this needs measurements between the MS and three different BS). The calculation of the time distance between sender and receiver is made with timestamps.
TDOA on the other hand makes use of multiple synchronised BS. These measure the difference in time of arrival of the same signal at different stations. With this information, it is possible to estimate the position of the MS. (Kolodziej et al. 2006, p. 102f)

2.3 Localisation methods

There are multiple ways to estimate the position of a mobile station. As already briefly stated in the introduction, three main approaches are common: proximity, lateration (power- and time-based) and fingerprinting.
In the description of the approaches, base stations (BS) are nodes with known positions sending and receiving wireless signals. The position of a mobile station (MS) needs to be estimated. Due to the similar nature of all the concepts, similar problems arise. To reduce repetition, the common challenges will be discussed first and later the approaches will be explained in detail. Further challenges will be listed, where applicable.

2.3.1 Overall challenges

Overall challenges arise mainly in three fields: the gathering of information, the geometry of the base stations and the difference in the base stations.

Gathering information

Due to the fact that signal measurements (RSSI) are not collected continuously, all the approaches suffer from the technical problem that there might not be enough measurements (of sufficient quality).
This leads to the fact that depending on the BS used, real-time measurements might have a delay of multiple seconds. Moving speed matters as well. Slower or even stationary targets can be estimated more precisely, since more readings are available (Huang et al. 2011, p. 331).

Base station geometry

All approaches that need three or more antennas are vulnerable to the layout of the BS, i.e. the geometry. Not all triangles yield similar accuracy. Studies show that wheel-like deployments (Yang et al. 2009) yield better results. The result of an evaluation of BS-layouts can then be part of a quality assessment of the nodes. This assessment can later be used in triangulation leading to more accurate results (Yang et al. 2010).
Another known effect of insufficient BS-layouts is called "string of pearls", where BSs are situated besides a straight road (therefore in a straight line), thus making triangulation impossible due to the fact that the positions and the resulting vectors from MS to BS are linearly dependant (Wigren 2012, p. 426).

<u>*Different output of different BSs*</u>

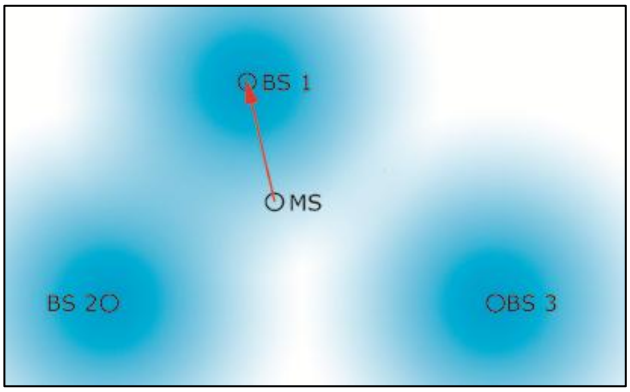
Different signal strength readings due to different antennas (outputs, gains, radiation patterns) need to be taken into account. This can be solved by pre-calibration or by estimating the offset factor of the signal strength by comparing BS information (Beder et al. 2012, p. 2ff).

2.3.2 Proximity

The proximity approach is relatively simple to implement. It is based on the introduced physical fact that a wave loses signal strength while passing through space (see 2.2.1). Hence, a stronger signal indicates a nearer BS, assuming all BSs send with equal power. With this approach the strongest received signal can be selected, the estimated position is the position of the corresponding BS (Hui et al. 2007, p. 1071). As simple as this approach is, it can easily be seen that it results in imprecise estimations. Unless the MS is situated at exactly the coordinates of the BS, the "calculated" positions are always wrong, the error margin depends on the distance to the BS (the nearer the better) and on the existence of non uniformly distributed obstacles (e.g. occluded nodes). Systems using infrared radiation (IR) and radio frequency identification (RFID) as well as cell identification in mobile networks are based on this method (Hui et al. 2007, p. 1071).

Fig. 1 shows the setup of three base stations and one mobile station. The signal strength at a given point is indicated by the blue circles. The strongest reading is recorded by BS1, therefore the estimation of the position of the MS is equal to the position of BS1.

Figure 1: Proximity



Source: Illustration by author

<u>*Challenges using proximity*</u>

Due to the nature of the proximity approach, the main challenge is the fact that the positioning performance can only be improved by adding more BSs in the same area. The denser the distribution of the nodes, the lower is the location error of the algorithm. Still, in applications, where precision down to the meter is not required, or where the strongest BS is sought (e.g. cell identification), this approach is preferred due to its simplicity.

2.3.3 Lateration and angulation

When using signal strength and time delay information, two basic approaches to estimate a location exist: lateration and angulation. **Lateration** is defined as the location determination from multiple distance measurements. The use of angle or bearing data relative to points of known position to find a targets location is called **angulation** (Hightower et al. 2001, p. 57). The process itself is called **triangulation**, referring to the usage of triangle geometry. Triangulation can be done in multiple ways.

Lateration

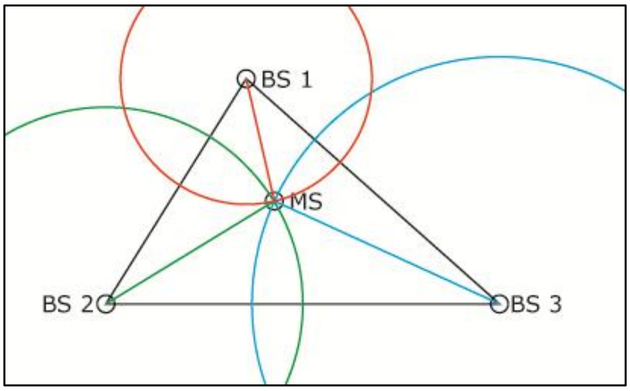
In wireless networks, lateration is a method of determining the position of the wireless device as a function of the lengths between the wireless device and each of the BSs.

In two dimensions (e.g. if only one floor of a building is analysed), lateration requires at least three signal strength measurements from three different BSs to pinpoint the location of the wireless device. This approach is called trilateration. The distance from each sensor is determined by making the assumption that the device (with a given signal strength) is at a certain distance from a sensor based on circular coverage maps. The circles of the BSs overlap leading to an estimated location of the wireless device. If no precise intersection can be found (e.g. due to attenuated signals) the coordinates where the signal strength circles overlap can be used to calculate an estimation (e.g. by using a centroid approach). If the circles do not overlap, lateration cannot find an estimation.

Trilateration of uniform radiating RF-signals (circles) from all devices has an error rate of +/- 6.1 meters. (Kolodziej et al. 2006, p. 149)

Fig. 2 shows the setup of three base stations and one mobile station with respective RSS-readings (circles in red, green, blue). The intersection of the circles marks the estimated position of the mobile station.

Figure 2: Trilateration



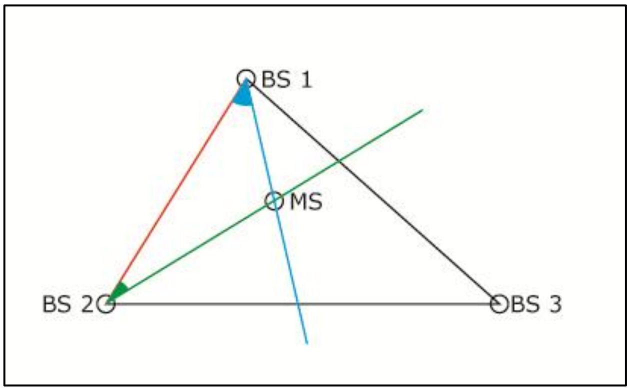
Source: Illustration by author following Bensky (2008) p. 8

Angulation

Angulation (often called triangulation as well) uses angles to determine the location of an object in space. In two dimensional space, angulation needs two angle measurements and the distance between the two measurement points to calculate a third point in space. (Kolodziej et al. 2006, p. 149)

Fig. 3 shows the setup of three base stations and one mobile station. The distance (red) and two angles (blue, green) are used to locate the mobile station.

Figure 3: Triangulation



Source: Illustration by author following Bensky (2008) p. 9

Challenges using angulation and lateration

The main challenge of angulation and lateration approaches is the calculation of distances from RSS or TDOA readings. In the RSS case, the more relevant factors can be included in the propagation model, the better the estimation. This can lead to using multiple algorithms via Markov-Models for different days, hours a day, different locations or a multitude (see Abbas 2012, p. 7). It is obvious that developing site specific propagation models and collection the necessary information is a time consuming step that, given the fact that the surroundings are rarely static and systems change, has to be done over and over.

When using TDOA and TOA, another big challenge is the synchronization of the nodes. Given that the BS can have feedback from the MS (may require additional signalling), this can be done by synchronizing the nodes with the GPS clock of the MS (Lami et al. 2013, p.2f). Furthermore, in TOA the MS has to be synchronized and the sent signal has to be labelled with a timestamp (Bensky 2008, p. 30, Liu et al. 2007, p. 1068f). Since the speed of an electromagnetic wave equals the speed of light, measurements have to be very precise. Small errors in measuring time lead to big errors in localisation (1 nsec of timing error equals 0.3 m of location error). This might be a bigger problem on small scale environments (indoor) where the proportional error is accordingly larger.

For angulation main challenges arise from the fact that the angle of the arriving signal cannot be measured exactly. The uncertainty leads to not estimating a point, but an area with size depending on the precision of the distances between the two BS, precision of angle measurements, furthermore the angles themselves (acting as an error multiplier) and the distance between MS and BS (Bensky 2008, p.189ff).

2.3.4 Centroid

The centroid approach also makes use of RSSI readings or TDOA-information, it is however different from the angulation and lateration approaches and rather counts as an improved proximity approach. The centroid of a triangle (or any form with a defined shape) is the centre of mass. In a triangle the centroid is the point of intersection of its medians. A median originates at a corner and divides the triangle in two equal shapes.

The centroid algorithm is considered one of the simplest positioning algorithms since it relies on very basic geometric calculations with the positions of an arbitrary number of base stations. The position of each BS is evaluated during a training phase. Using this map, the algorithm calculates the position of the mobile station by “computing an average of the estimated positions of each of the heard BS's”. (Kolodziej et al. 2006, p. 151)

Hence, the general calculation of the centroid point (CP) for a number of n BSs, where BS_i stands for the coordinates of the i-th BS, is as follows:

$$CP = \frac{BS_1 + BS_2 + \cdots + BS_n}{n}$$

Since this formula always leads to the same centroid point for given BSs, the localisation of the mobile station would be wrong in most cases (the cases where the MS is not exactly at the CP of the BS's).

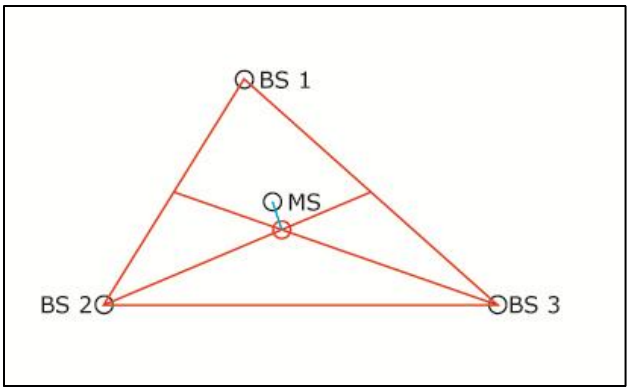
Therefore, more complex versions weight the positions with RSS during the scan. Using three BS, with the **weighted approach**, three RSS readings between MS and each BS result in three vectors. These are used to bridge the supposed distance between CP and MS-location. Hence the formula to calculate the CP changes (Blumenthal 2007, p. 3):

$$CP = \frac{\sum_{j=1}^{n} (w_j * BS_j)}{\sum_{j=1}^{n} w_j}$$

With w_j being the weight function between the MS and BS_j. w_j can be derived from the RSSI-reading, a quality indicator, or a function thereof.

Fig. 4 shows the newly calculated centroid point in red, with the blue line indicating the result of the weighting by RSS-Information.

Figure 4: Centroid



Source: Illustration by author

Other geometrically fixed points in a triangle could be used as well. Candidates include the known incenter and circumcenter, but a multitude of distinct points on a triangle are known (Kimberling 2013). As each fixed point potentially brings different pros and cons for an estimation starting point, the pitch for the best initial candidate would be beyond the scope of this thesis. Key arguments for the centroid point are the following facts:

- The centroid is the arithmetic mean of all the BS-positions;
- The centroid always lies inside the triangle (this is a problem, if the circumcenter is used).

Challenges using centroid

Similar to the proximity case, one challenge of the centroid approach is the standard error. Since multiple readings (at least the three strongest) need to be taken into consideration, the standard error is supposedly bigger than in the case, where the nearest neighbour is chosen. This is due to the fact that points outside the half distance between centroid and each BS are better estimated by using the BS position. Again, with a denser network of BSs, localisation by centroid gains in accuracy. The standard error can further be minimized by using an approach, where the triangle made up by three BS is split in smaller triangles. Depending on the signal strength information, one of these triangles is selected, and it's centroid is estimated to be the location of the MS (Liu et al. 2012).

Together with the fact that the centroid point is always inside the triangle, cases at the border of the network are not accounted for[6]. Furthermore triangle geometry and density of the BSs

[6] e.g. the MS might be situated outside the area encased by BS's

plays an important role. Using a centroid approach, different algorithms therefore yield better results in different situations. Where multiple nodes are available (in the centre of the net), other techniques can be used compared to the case where only isolated nodes are available (border of the net) (Liu et al. 2012, p. 3ff). Still a reliable test whether the MS remains inside the triangle or outside plays a big role for satisfactory estimation and is one part of this thesis's contribution.

Another factor arises, when using a weighted centroid approach. Here choosing the weights becomes a critical part and a further source of error. Possible candidates are RSSI or functions thereof (proportions, differentials, adaptation by using propagation models) or quality indicators (LQI, or measures accounting for triangle geometry).

2.3.5 Fingerprinting

Fingerprinting techniques relay on databases containing previously gathered MS positions and corresponding signal strength readings to multiple BSs. New RSS-readings from a MS are then compared to the database, resulting in an estimated position calculated from comparisons with the available data (Bensky 2008, p. 2). Thus fingerprinting has to happen in two steps (Liu et al 2007, p1070, Koweerawong et al 2013, p. 412):

1. **Offline** or **training stage**: signal information is gathered at reference points[7]. Often the data is interpolated between the antennas or polygons of regions with similar signal information are calculated (Wigren 2012 p. 426).
2. **Online** or **positioning stage**: measurements on a handset are compared to the information gathered in step 1, a position is returned.

There are different algorithms based on pattern recognition techniques used in step 2 to locate the position of the handsets. These are similar in the workings and include (all Liu et al 2007, p 1070f):

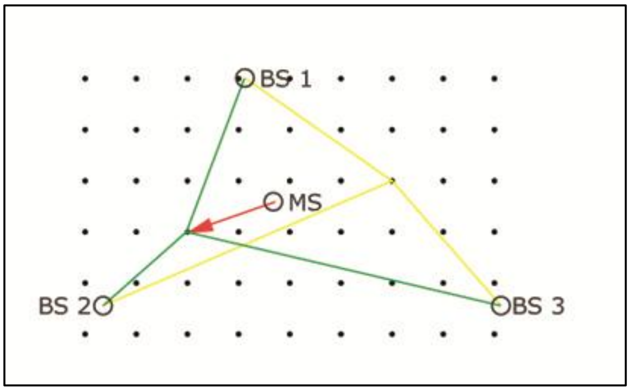
- Probabilistic methods and k nearest neighbours (kNN, Koweerawong et al, 2013, p. 412f) are similar approaches, where certain locations are more probable than others given the signal readings. This can be done on per BS basis (probabilistic method) or by comparing all records and picking the closest k (kNN). Like that, the overall likelihood of one or multiple location candidates can be derived from the information available. The estimated location of the MS can then be interpolated from the known positions of the most likely BS.

[7] Due to the nature of step 1, fingerprinting techniques are often called "scene analysis" (examples: Liu et al 2007, p 1070).

- Neural networks and support vector machine-approaches (SVM) make use of machine learning theory. During stage 1, signal information and known positions are added for training purposes resulting in appropriate weights for location estimation.
- Smallest m-vertex polygon (SMP) is a geometric solution. In signal space, M polygons are formed by choosing at least one candidate with matching RSSI from each BS (each BS can have multiple readings). The coordinates of the vertices of the smallest polygon are then averaged, this gives the location estimate.

Figure 5 shows a fingerprinting situation with three BS and one MS. The smaller dots indicate fingerprints. The red arrow indicates the location of the matching fingerprint (indicated with green lines signifying measured signal strength) from the database - this is where the MS is expected to be. The yellow lines indicate a different fingerprint with non matching signal strength measurements.

Figure 5: Fingerprinting



Source: Illustration by author

Challenges using fingerprinting

Due to the different setting of fingerprinting approaches, technical challenges remain and further challenges arise.

Again the number of BS plays a leading role. Most algorithms perform better when more BS readings are available (Huang et al. 2011, p. 330). Since some algorithms compare combinations of readings over multiple BS, a dropped or invisible node in stage 2 leads to localisation errors (Beder et al. 2012, p. 2). Another important role of the number of BS is the fact that the accuracy of a fingerprinting map solely depends on the BS-gradient. It is therefore possible to derive this information in advance based on the fingerprints only (Beder et al. 2011, p. 6).

A further major challenge is the fact that the value of the information collected in stage 1 is diminishing over time. Due to the fact that environments change, the information has to be collected constantly, some algorithms therefore work with constant feedbacks to generate the fingerprint (Koweerawong et al. 2013, p. 414).

Furthermore, while collecting reference points in a building, the person holding the MS has an impact on the RSSI-readings obtained as pointed out in Chapter 2.2.3. It is possible however, to adjust the readings to account for this factor. (Fet et al. 2013, p. 504ff)

To ease the creation of fingerprinting maps, some authors propose the usage of simulation. Simulation can be done empirically (by using propagation models) or physically (by using similar approaches to ray tracing). With the new approaches new sources for errors in different fields arise: the main problem might be inaccurate geographical databases (e.g. wrong placement or height of buildings, wrong placement of antennas) leading to wrong fingerprints and again wrong estimation. The advantage of simulation however is the fact that not only outdoor fingerprinting maps, but maps for indoor positions could be simulated given accurate models accounting for different settings (e.g. walls, furniture and buildings). Still, this approach is too inaccurate to be used for localisation in dynamic environments. (Freedman et al. 2012).

2.4 Dynamic approaches

It has already been pointed out that RSSI measurements change due to changing circumstances. It is therefore important to note that static RSSI based localisation approaches in indoor environments have their limitations. Since the approaches to calculate path loss are not built to change dynamically, adjustments are needed.

Multistate model

To change between different path loss functions, Abbas et al. (2012, p. 6f) propose the usage of a multistate-model that adapts according to changing factors like the number of moving people. This however needs great and constant effort to account for different settings as has been discussed in the previous chapters.

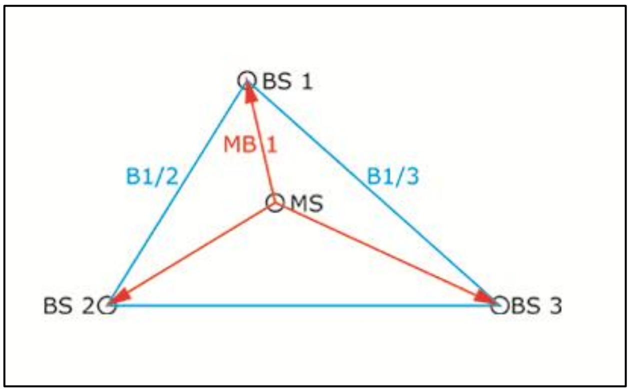
Reference-nodes

Another attempt is given by Kim et al (2011, p. 934f), where the authors propose to cross-monitor base stations to further improve localisation by getting information about the environment. Cross-monitoring is an approach, where the RSSI between BS is measured and compared to expected values to gain information about the environment. It can be done continuously, thereby allowing for dynamic adaptation of expected path loss between the mobile station and the base stations.

Dynamic adaptation factors are obtained by calculating a loss factor between either reference nodes and a BS or between BSs (as done in this thesis). As the position of reference nodes and BS are known, free space path loss can be calculated from the distance. The difference to the path loss received in the measurements results in a environmental factor accounting for everything blocking or changing the signal between the BS. This factor can then be used to weight the signal strength information between MS and BS.

Fig. 6 visualizes the weighted centroid approach using BSs as reference nodes. Here the RSSI MB1 would be weighted according to the signal strength measured on vectors B1/2 and B1/3 before weighting the centroid.

Figure 6: Weighting of Base Stations by RSSI



Source: Illustration by author

3 Analytical discussion

From the triangulation approaches presented in the previous chapter, the weighted centroid is the most practical for different situations as no propagation formulas need to be calculated. Weighting will be done with MS to BS RSS measurements adjusted with inter BS RSSI measurements (BSs used as reference nodes).

The following sub chapters will introduce the necessary calculations to estimate the MS position. Challenges arising due to the simplicity of the approach will be addressed. This includes calculating base values for weighting the centroid, collecting inter BS measurements, analyzing MS positions on the triangle, testing for the outside-case and finally adjusting the weighted centroid algorithm.

3.1 Weighting the centroid

As previously shown, weighting the centroid can be done in multiple ways. As the goal is to define a low maintenance weighted centroid algorithm, only the information available will be used. This information consists of the positions of the BS, the RSS-readings of the MS and the inter BS measurements[8]. As signal strength fades with distance, the usage of the latter poses the problem that a change in distance has to be converted to a change in RSS by means of a path loss formula. This leads to the necessity to estimate a path loss exponent depending on the environment in situations, where free-space path loss cannot be used.

3.1.1 From RSS to weight

Weighting the centroid is based on the idea that the MS tends to be nearer a BS where a stronger signal is detected. A first problem while using any data obtained by sensors is the granularity of the information available. In a perfect situation, every moment would have a precise RSS information of all available BSs. Unfortunately this is rarely the case and moments have to be aggregated to get a complete picture of the situation. Since a MS is mobile, the movement speed of people has to be taken into account. An observation of the moving speed of the author yields a speed of around 1m per second. Given the available data and the estimated moving speed, an upper limit of the measurement-moment can be defined. Smaller durations are generally more desirable in real-time positioning because aggregating over long durations decreases the frequency of obtaining estimates. 3-5 seconds appears to be a good compromise between too short to get enough measurements and too long for precise estimation. However, if only sparse information is available, the estimation with stationary targets becomes the only feasible situation with reliably high accuracy.

[8] These then provide the only available information about the environment, e.g. walls or other signal changing factors.

If multiple values are available within a measurement-moment, aggregating these to a single RSS value needs to be addressed next. As attenuation might change in the defined timeframes, multiple statistics can be argued to give a meaningful result. Examples include minimum and maximum, average and median (both normal and trimmed), the latter being more robust than the former. As no best practice is known to the author, the maximum signal strength of each timeframe will be used in the calculations assuming a best performance and therefore best estimate.

Using RSS-information poses a further problem as the RSS is attenuated due to multiple factors already introduced in the previous chapters. RSS is not diminishing linearly with distance. Since moving the centroid is done in "distance-space" opposed to "RSS-space", conversion is a necessity. With identical BS assuming free space propagation, conversion can be done by using the following formula,

$$distance = 10^{(RSS/N)}$$

where N/10 is the path loss exponent. The gained distance estimates could now be used for weighting. It is however to be noted again that RSS not only depends on the distance and the path loss exponent, but also on the occurrence of attenuating factors like people or walls. To account for these factors, inter BS measurements can be used for weighting.

3.1.2 Inter BS measurements

As the distance between two BS is known to the system, the free-space path loss between the two can be calculated. Pitching these figures against the available inter BS measurements yields information about the situation between two nodes. Recalling Figure 6, two inter BS-readings can then be averaged to estimate an attenuation-factor between the BS and the centroid point.

The duration of the measurement-moment and the aggregation of the recorded data are again the main problems that need to be solved. The first challenge becomes greater, as now readings between all BS need to be collected too. The period over which the inter BS data is collected depends on the factors one wants to isolate (shadowing, fast fading). At least the BS's with the strongest MS readings need to have inter BS information. A complete net is desirable as the nearest neighbour BS can change depending on the MS's real position and mobility.

3.2 Analysis of positions on the triangle

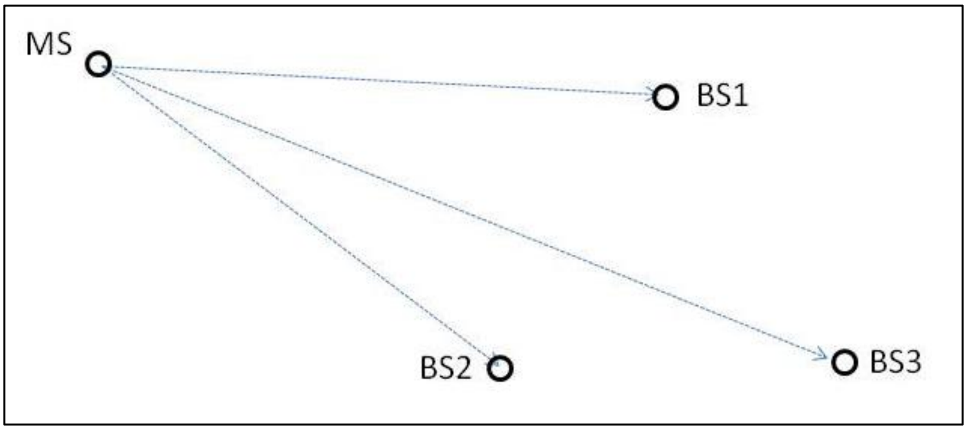
As already mentioned, the weighted centroid approach might lead to problems, if inter BS-measurements are used to weight the RSSI-readings and the actual position of the MS lies outside the triangle. In this case, at least for one signal reading, a wrong weight will be calculated. Furthermore the outside situation leads to an initial error that cannot be corrected by means of conventionally weighting the centroid.

To clarify the necessity of an inside/outside-test, the author will present multiple situations on the triangle. It is to be noted that without inter BS signal strength information, in most cases, it might be hard to determine, whether the MS lies inside the triangle or outside. Since not all systems allow for inter BS measurements, the author considers both situations, with and without inter BS information. Furthermore line of sight and attenuated signal situations are distinguished. For an overview, see appendix II.

3.2.1 Case 1: LOS/NLOS weak signal

If the users signal is weak at all BS and/or the BS are far from each other, it is not possible to distinguish whether the position of the MS lies inside or outside the triangle, due to fluctuations of the RSSI at lower values. Absolute values can be used to set thresholds, but these are sensitive to actual propagation conditions (e.g. obstacles) and to variations in MS transmitting power (different models, layouts, attenuation). If inter BS readings are definitely stronger than the weak signals received from the MS, the BS can be positioned outside the triangle.

Figure 7: LOS/NLOS weak signal

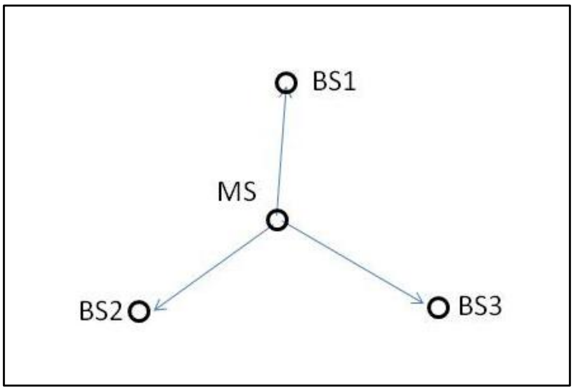


Source: Illustration by author

3.2.2 Case 2: LOS with similar signal strength

In this scenario, all BSs receive a signal with similar strength from the MS. If the signal is considerably strong a position of the MS inside the triangle can be expected. The more similar the signal strength, the higher is the probability that the MS is inside the triangle. Inter BS-readings lead to better estimates by serving as reference signal strength to be expected inside the triangle.

Figure 8: LOS with similar signal strength

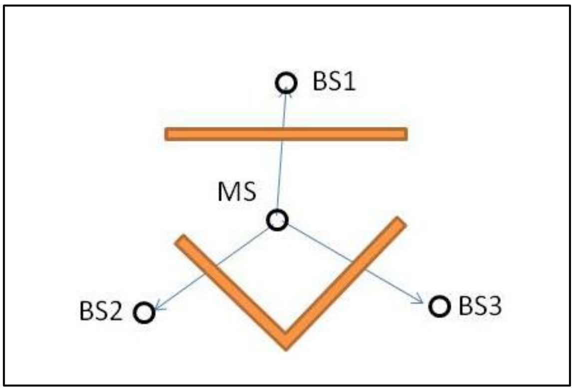


Source: Illustration by author

3.2.3 Case 3: NLOS similar strength

Without line of sight between the MS and one or multiple BS different further problems arise. Now one or more BS readings might be weaker due to attenuation. If all signals are blocked equally, the situation is similar to case 2 discussed previously. Since attenuation is weakening the signal, the situation, where a precise estimation is hindered by low signal strength, will be encountered more often. Again, inter BS-readings can help as reference to estimate the readings from the MS, thus making it possible to position the MS inside or outside the triangle.

Figure 9: NLOS all blocked

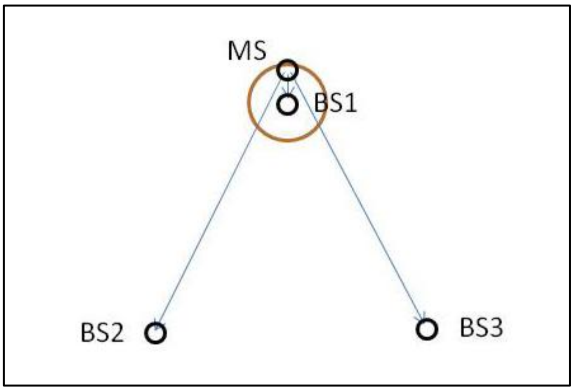


Source: Illustration by author

3.2.4 Case 4: LOS with different signal strength

If one signal is strong and two signals are weak, a position in vicinity of the BS (marked with an orange circle in Figure 10) receiving the strong signal can be expected (proximity approach). Due to fluctuations of RSSI, it is not possible to estimate whether the MS is inside or outside the triangle. With using inter BS information this estimation is possible depending on the signal strength received from the BS and the inter BS readings.

Figure 10: LOS different signal strength

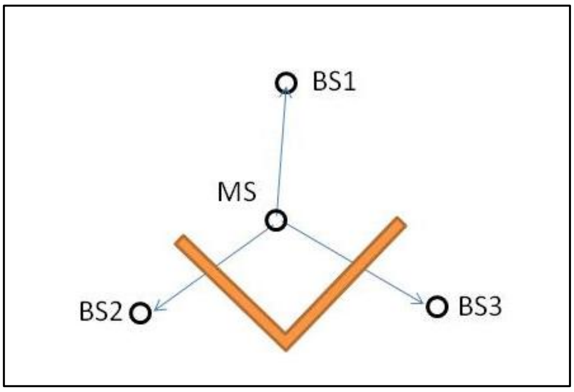


Source: Illustration by author

3.2.5 Case 5: NLOS with different signal strength

Now only the signals between the MS and some of the BS are attenuated. Hence again some signals are stronger than others. Since not all signals are attenuated, the situation cannot be distinguished from the ones depicted in Figure 8 or 10, without the usage of inter BS measurements. The inter BS measurements can indicate the attenuation of the MS to BS signals only if the inter BS signals are attenuated by the same obstacle as the MS-BS signal. This is the case if the attenuation is caused by a wall or a similar structure enclosing the MS (e.g. BS situated in another room). However, attenuation caused by a person shielding the MS from the BS cannot not be detected, since the inter BS connectivity is not hindered.

Figure 11: NLOS/LOS mixed situation



Source: Illustration by author

3.3 Advanced geometric approaches

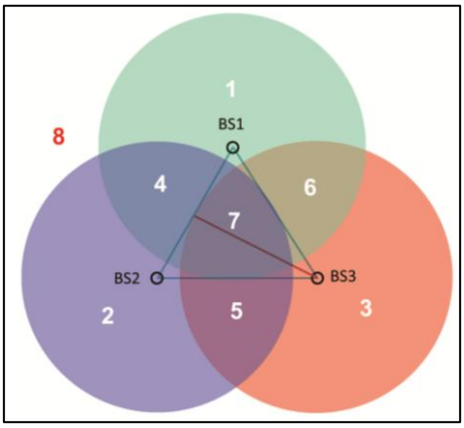
With the possible situations in mind, the author will focus on advanced approaches on inside/outside-tests. While the last section highlighted situations, where it is possible to estimate if the MS is inside or outside the triangle, the following approaches will add further information indicating approximately where the MS might be positioned (e.g. the MS might be outside BS A or outside the line segment between BS A and BS B).

This section will not discuss optimisation of the inside case, as the final positioning will be done by weighting the centroid (this could however also be improved by subdividing the triangle[9]). The following subchapters introduce three approaches to the inside/outside-question. The different approaches need different information and depend on different factors that might perform better under different circumstances and can be divided in two groups: approaches where distances need to be calculated from RSSI (and vice versa) and approaches where solely RSSI-readings are compared.

Since distance based approaches rely on propagation formulas, the calculation of distances from the RSSI-values might contradict the idea of using a centroid approach to eliminate the need of such formulas. However, under certain circumstances, these approaches might work well enough and lead to more precise inside/outside estimations.

With three BSs, three signal strength or distance measurements from the MS can be drawn as circles centred around the corresponding BS. Three overlapping circles lead to a maximum of seven possible areas, where the MS could be located on the map. If a reference signal strength is used as threshold (e.g. only use MS readings stronger than an inter BS measurement), the three circles have defined boundaries. Thereby another (8th) area is created: the outside area with unknown position results. Figure 12 gives an example, where the distance to (or the signal strength at the) opposite line segment is used as circle radius. the seven areas are coloured (numbers: 1-7), the eight area remains white (8).

Figure 12: Eight areas on the triangle



Source: Illustration by author

[9] As in Liu et al. 2012, where triangle geometry is used to subdivide the triangle in smaller segments thus leading to multiple sub centres.

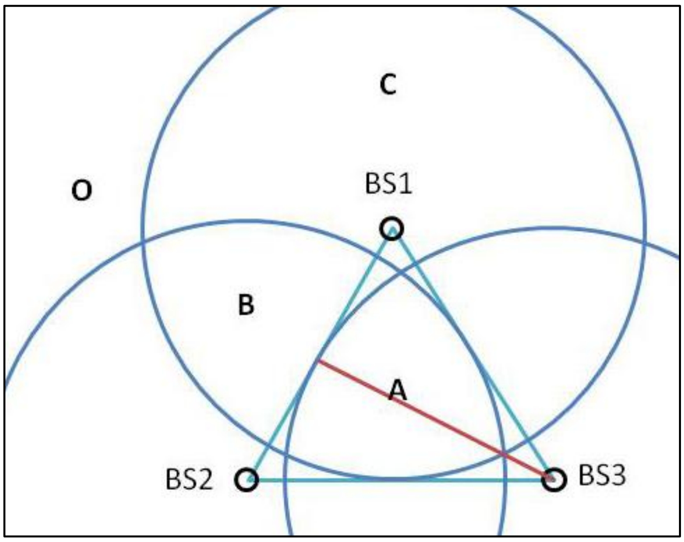
The overlapping areas can be calculated from the readings by using the standard AND operation from set theory. Here "inside circle around BS1" AND "inside circle around BS3" results in the area between the two.

3.3.1 Approach 1: expected RSSI calculation from calculated distance

In the first approach shown in Figure 13, the distance (marked in red) between BS3 and the opposite line-segment is calculated by means of geometry. This is possible, because the coordinates of the deployed BS are known. This distance is then converted to an expected signal strength by means of a propagation formula yielding the minimal strength BS3 can receive from the MS, such that the MS is inside the triangle. As this is done with all three BS, the result are again eight possible areas previously shown in Figure 12.

Due to the construction with the distance to the opposite line-segment, area *A* is always inside the triangle[10], *B* and *C* are mostly outside the triangle but have a position in a known area and *O* marks the area outside the triangle where MS positions will be unknown. Unless the resulting position is at *O*, the expected position of the MS can thus be narrowed down. The challenge of this approach lies in adjusting the propagation formula. Collecting and using information about the site, surroundings and obstacles is a must.

Figure 13: Inside/outside a1



Source: Illustration by author

In the following example, S_BS_i stands for the signal strength gained from the calculation of signal strength S at the position at the opposite line-segment of BSi. S_MS_i stands for the signal strength of the MS received by BSi. "$A < B$" stands for "signal A is weaker than signal

[10] assuming perfect calculation of signal strength from distance

B" or "signal B is stronger than signal A". Example: If $S\_BS_3 < S\_MS_3 \cap S\_BS_2 < S\_MS_2 \cap S\_BS_1 < S\_MS_1$, the position of the MS lies in the area marked with an A.

3.3.2 Approach 2: RSSI-adjustment from calculated circular segment height

In a similar second approach, again the distance between BS 3 and the opposite line-segment is calculated resulting in distance *d* as shown in Figure 14. Depending on the triangle geometry, two circle segments can be constructed (as two circles with radius $d_{3,1}$ and $d_{3,2}$ can be drawn around BS3). Figure 14 shows both, the easy case, where the distance between BS3 and BS1 ($d_{3,1}$) is similar to $d_{3,2}$ and the more complex one, where $d_{3,1}<d_{3,2}$.

Now, the height of the circular segments *h* is calculated by using the $d_{3,1}$ and $d_{3,2}$. If the triangle formed by the BS is isosceles or equilateral (having two or more equal sides), leading to the both distances $d_{3,1}$ and $d_{3,2}$ being equal, it would be sufficient to only calculate one segment height. Otherwise, each circle will have a different height *h* to the line BS1-BS2.

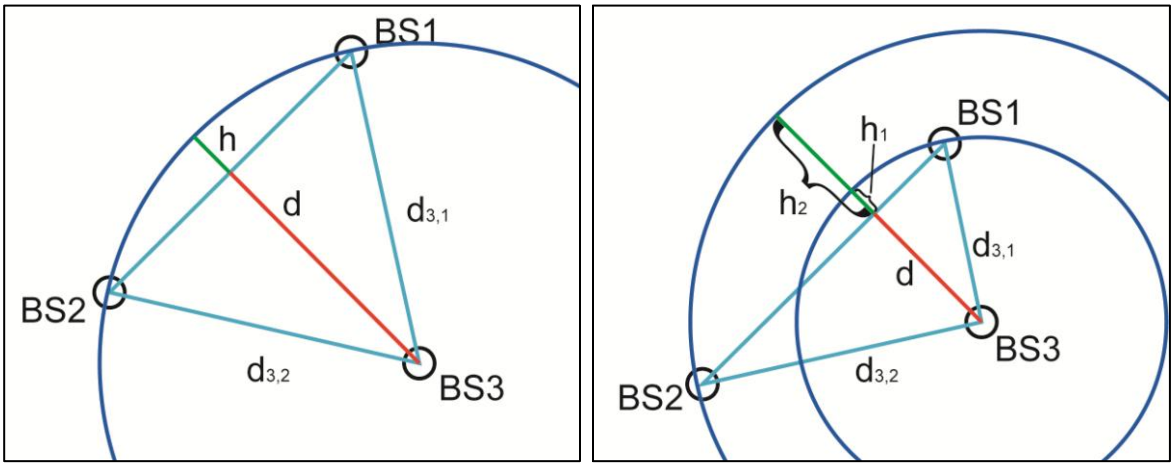
Both distances can be added (resulting in $d + h$). The fraction of *h* on the total distance ($d + h$) is the part being outside the triangle. With two different *h*-factors the smaller/bigger one or an average between both might be considered for the further calculations.

The fraction $h/(h + d)$ can now be used to adjust the inter BS readings from BS 3 to the corresponding BS, resulting in the expected signal strength at BS3+distance *d*. Since the inter BS readings adjust for the environment, this reading is automatically adjusted[11] as well and can now be compared with the received signal strength from the MS.

The procedure must be repeated for every BS, resulting in similar 8 possible areas, where the MS could be located as previously shown in Figure 12.

[11] Assuming uniform distribution of obstacles or a LOS situation.

Figure 14: Inside/outside a2



Source: Illustration by author

Since the relationship between signal strength and distance is not linear, the weight-factor gained cannot be applied to signal strength directly and must thus be converted to a distance. This however results in the need to employ propagation formulas. Since attenuation through obstacles is already accounted for in the inter BS measurement, the author expects a fairly simple propagation formula, where only the path loss exponent needs to be estimated, in order for the formula to be sufficiently precise.

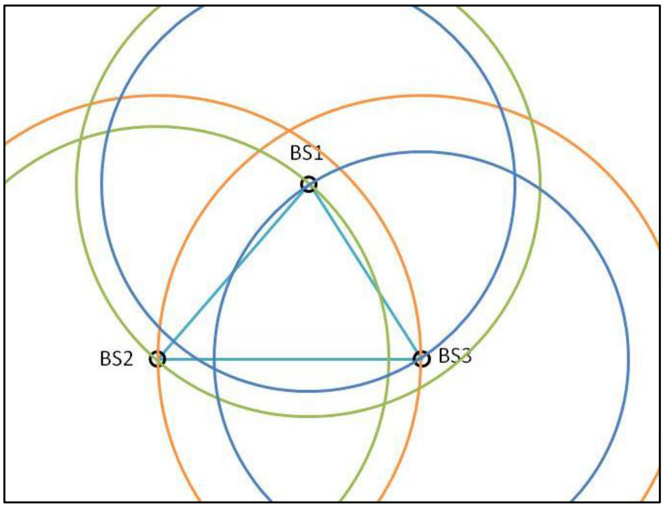
An example will not be given, as the situation is similar to the one in Figure 13, only the signal strength at the opposite line-segment of the base station in question is calculated differently.

3.3.3 Approach 3: Multi-zone RSSI

In the third approach, the RSSI readings between MS and BSi (S_MS_i) are each individually compared to the inter BS-readings ($S\_BS_{i,j}$ stands for the reading between BSi and BSj). If S_MS_x and S_MS_y are stronger than the $S\_BS_{x,y}$, the MS can be estimated in the segment formed by the circles around BSX and BSY with radius equal to the distance between BSX and BSY ($d_{x,y}$).

If this is done for all BSs, the areas defined by six overlapping circles can be used to estimate an improved starting-point for the weighted centroid-approach. Depending on signal strength and triangle geometry, this approach can lead to a calculated "inside" area that is actually bigger than the triangle formed by the BSs themselves.

Figure 15: Inside/outside a3



Source: Illustration by author

Figure 15 shows a situation with different inter BS-readings (again the distance between two BS is taken as approximation). $S\_BS_{2,3}$ is coloured in orange thus a signal-strength/distance circle in orange is drawn around BS1 and BS3 (as both share $S\_BS_{2,3}$). Similar for the other BSs, $S\_BS_{1,3}$ is marked in blue and $S\_BS_{1,2}$ is marked in green.

Since each BS has two neighbours, two distances can be used in evaluation. These can be averaged or the minimum or maximum can be taken depending on the situation. If

$$S\_BS_{1,3} < S\_MS_1 \ \cap \ S\_BS_{1,2} < S\_MS_1,$$

the estimated position lies within the blue circle with radius $DBS_{1,3}$ around BS1, as $RBS_{1,3}>RBS_{1,2}$. However, even if using both estimates and switching depending on situation might lead to more precise estimation overly complex situations are created while trying to narrow down the estimated position of the MS as the following examples show: If

$$S\_BS_{1,3} > S\_MS_1 \cap S\_BS_{1,2} < S\_MS_1,$$

the estimated position lies between the blue and the green circle around BS1. The area in which the MS is estimated thus becomes ring-shaped (*ring1*). If furthermore

$$S\_MS_3 > S\_BS_{1,3}$$

(MS is also inside the orange circle around BS3 with radius $d_{2,3}$), the intersection is the U-shaped segment of ring1 near BS3 (*segment1*). If we now look at segment1, the best estimate of a MS position would again be the centroid. Both last examples (*ring1* and *segment1*) are shapes having the best estimate outside the segment area. Hence either an average, minimum or maximum approach between the two available readings per BS is suggested.

3.4 Evaluation of advanced geometric approaches

The complexity of the situation changes when triangles are not nearly equilateral anymore. Furthermore added walls between the different links attenuate the signal. To evaluate multiple situations, the advanced inside/outside-tests introduced in the previous chapter have been modelled in Matlab.

3.4.1 Model overview

The Matlab model consists of a basic setting with 3 BSs and no to multiple walls. Positions of MS can be input manually or generated randomly. For every position, the algorithm first calculates inter BS values and RSSI readings at the BSs. In a real world application, this data would be obtained from BSs.

The Inter BS values are calculated using a reduced version of the log-distance path loss model with wall attenuation factors introduced in Chapter 2.2.4. (Sarkar et al. (2003, p. 58). As the modelled case is a two dimensional one, floor-attenuation factors are not part of the equation. The wall attenuation factor was set to -9 dB per wall[12]. A path loss of < -100 dB is adjusted to -100 dB signifying total loss of signal. N, the path loss-exponent was set to 2.7, a common value for office situations in Wi-Fi-Networks.

The system now calculates, if the MS is located inside or outside the triangle. This is done in two ways. In the first step a geometric evaluation is done. Here it is calculated whether the position lies geometrically inside the triangle formed by the BSs. This test always generates a correct answer and can thus be used as a benchmark to evaluate the different algorithms. In the second step, one of the proposed algorithms is actually run. The algorithm independently calculates if the MS should be estimated inside the triangle, depending on the inter BS values and the RSS-information available from the analytically generated values.

If both, the geometrically and the algorithmically calculated results match, the position is marked in green, otherwise, a red mark indicates the no-match.

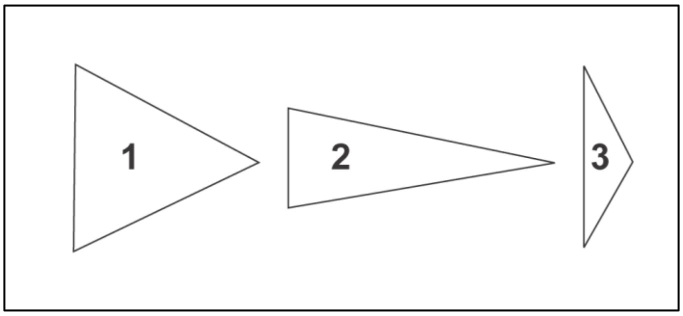
3.4.2 Evaluation setting

To pitch the algorithms against each other, multiple situations as show in Figure 16 have been evaluated:

1. basic nearly isosceles triangle,
2. pinched pointy triangle where two BS are near and a third one is further off,
3. pinched flat triangle, where one of the angles is > 90 degrees.

[12] For the simulation, this value can be chose at will. A greater WAF will result in stronger blocking of RSS.

Figure 16: Triangle shapes



Source: Illustration by author

Since inter BS values will be used to account for blocking walls in the final positioning algorithm, the inside/outside algorithms have been tested in different situations:

- BS triangles without a wall between BSs (LOS),
- BS triangles, where one of the edges was blocked by a wall (LOS between the other two),
- BS triangles, where two of the edges where blocked by a wall (LOS on only one edge on the triangle).

In the evaluation, the focus is on the algorithms introduced under 3.3.2. (termed inOutTwo, or short io2) and under 3.3.3. (termed inOutThree or short io3) in the last chapter. As already discussed in 3.3.3. either an average (io3Avg), a maximum (io3Max: choosing the stronger signal as threshold, more restrictive) or a minimum (io3Min: choosing the weaker signal, less restrictive) of both values can be used. in the evaluation, all three approaches are tested.

3.4.3 Inside/outside-simulation: Results

Table 1 shows results of the simulation with 5000 random points per experiment. The percentage given is calculated as

$$Matching = \frac{N(algorithmically = geometrically)}{number\ of\ experiments}\ [\%]$$

As the geometric calculation is always correct, the value stands for the percentage of correct inside/outside estimations. This percentage depends on the size of the area for the random sampling and therefore on the size and form of the triangle (as the area is calculated from the triangle coordinates). Furthermore the number of matches depends on the complexity of the situation (number and placement of walls). Therefore only the values in the same row should be compared.

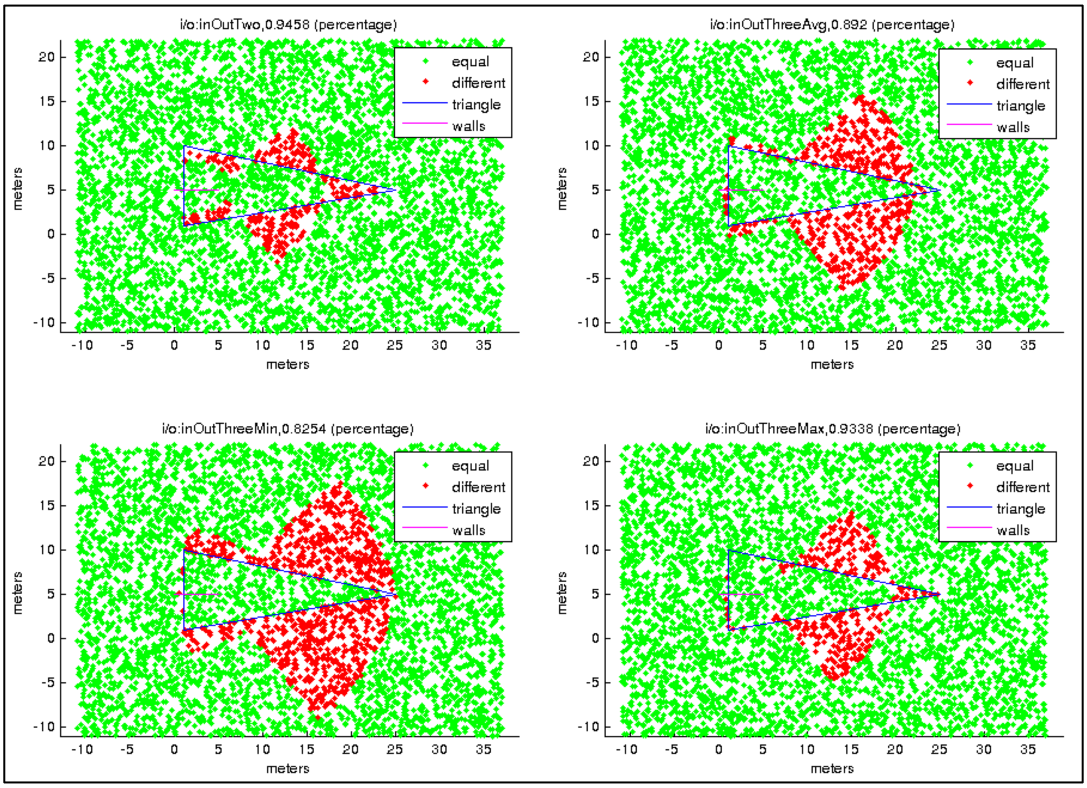
Table 1: Inside/outside simulation results

Matching % for N=5000	**io2**	**io3Avg**	**io3Min**	**io3Max**
11 Isosceles (I)	97.44%	91.82%	87.84%	94.72%
12 I, one link blocked	80.04%	66.98%	48.72%	87.4%
13 I, two links blocked	85.16%	77.34%	67.5%	92.58%
21 Pointy	95.76%	91.64%	68.22%	96.64%
22 Pointy, one link blocked	94.58%	89.2%	82.54%	93.38%
23 Pointy, two links blocked	89.02%	83.16%	62.44%	94.14%
31 Flat	94.62%	89.38%	71.42%	94.28%
32 Flat, one link blocked	90.48%	71.14%	65.08%	94.4%
33 Flat, two links blocked	90.56%	80.58%	56.04%	92.88%

Figure 17 shows the scatter plots for situation 22, pointy triangle with one link between BSs blocked by a wall. For the other plots see appendix III. Each plot stands for one of the algorithms. The triangle is formed by the BS's positions, a pink line indicates a wall, the dots (red and green) are the evaluated positions of a MS. Red dots mark positions, where the geometrical approach and the algorithmic approach disagree. This are thus the points, where the inside/outside test is wrong for the indicated algorithm. Hence, the more red dots, the worse the performance. Note that the number of correct estimates does not give an indication regarding the quality of the estimation. An example is a (fictive) algorithm that marks every position as always outside. This algorithm might have a better indication than the proposed ones (e.g. compared to Figure 17, inOutThreeMin), but the quality of the estimation would be unusable.

Assessing an estimation however is not a problem while looking at the plots, as some algorithms have constantly worse performance than others. This is the case with io3Min that is always worse than io3Avg, due to the fact that the weaker signal strength leads to a greater area indicated as "inside". io3Min is not a conservative algorithm as the inside-area is always estimated bigger than the triangle (unless severe attenuation occurs). On the other hand, io3Max tends to be too conservative in clipping the inside-portion. Averaging both leads to an improved version (io3Avg) compared to io3Min. Still the most restrictive of the three (io3Min) has overall the best performance, since the part that is estimated wrong remains significant even with the average.

Figure 17: 22 Pointy triangle, one link blocked



Source: Matlab output

This leads to the comparison of io3Max and io2. As the figures and the charts show, this comparison depends heavily on the situation. Depending on the case one or the other is more suited. The author assumes the io3Max to be more stable, as the calculation of the path loss from distance is a possible source of error in a real world scenario. A thorough pre analysis of the multitude of possibilities would however go beyond the scope of this thesis. Therefore both inside/outside algorithms will be implemented in the final algorithm using real world data.

3.5 Adjustments for estimated MS positions outside the triangle

After approximating the position of the MS with an inside/outside test leading to a first rough estimate, in a second step the author proposes to determine a better base of estimation of the target's position. As already pointed out, the centroid approach or its weighted version can only account for positions inside the triangle formed by the BSs (as the resulting point after weighting can never be outside the triangle and the centroid itself is always inside). In the following paragraphs, the author accounts for this fact by establishing a better base for estimation, if the position of the MS is supposed to be outside.

If the position of the MS is known to be outside the triangle, three options are available: ignoring, changing, adjusting.

The simplest option would be to ignore the available knowledge and start positioning the MS as if it would lie inside the triangle. This results in a greater standard error, as if a proximity approach would be used from the beginning. The algorithm could be changed though, if the MS is estimated outside the triangle, by using trilateration or a proximity-approach. Again different situations likely lead to different outcomes. The different enhancements would likely need additional information to the data obtained directly out of the network thus making the management of the system more complex. Furthermore minimizing the need for additional information was the goal of the approach advocated so far. This leads to the third option, adjusting, by improving the weighted centroid approach. One could argue that option two and three are fairly similar.

As the weighted centroid approach adjusts the estimated position of the MS from the centroid point towards the BS by means of vectors weighted by the signal strength and the inter BS measurements, these are the components that need be adjusted.

The first step to a better estimation is changing the base of estimation, i.e. the centroid point. As the inside/outside-test allows narrowing down the approximate area where the MS may be, the centroid point can be moved accordingly. Overall three different cases exist with the MS being outside the triangle as will be shown in the following subchapters. The goal is, to position the new base for the centroid algorithm at the centroid of the area, where the MS is to be expected. Second, depending on the updated position of the MS, the weights to apply on the improved centroid may have to be adapted. Furthermore, looking at the outside-positions, inter BS measurements might not be useful anymore as the resulting vectors from the BS to the improved centroid do not lie between two inter BS vectors, thus making it a challenge to adjust the vectors with the available inter BS readings[13].

Note that in the following Figures 18 and 19 just one example of triangle geometry is given. Signal-strength circles vary greatly depending on the shape of the triangle and the method used in calculating the threshold.

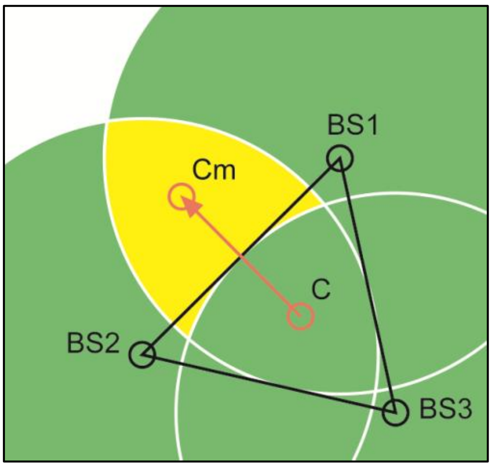
3.5.1 Case 1: "between", MS between two BS, but not the third

Figure 18 shows the "between" case, where the MS is located in the RSSI-circle around BS1 and BS2 but not in the circle around BS3. The MS thus lies in the shape coloured yellow. This shape appears similar to the inside-part of the triangle mirrored on the edge between BS1 and BS2. This is however dependent on the shape of the triangle and the diameter of the circles involved.

[13] Weighting the vectors for a centroid position inside the triangle still applies.

A new centroid base can be estimated by mirroring the centroid on the edge between BS1 and BS2 (resulting in *Cm*). Moving the centroid outside the triangle results in the fact that inter BS measurements cannot account for blockage between the nodes and the MS-position in all cases anymore. Furthermore, triangle geometry does not help in adequately adjusting the position of the new centroid point in the yellow area anymore, as no new outside reference point can be found. Mirroring the opposite BS does only work in equilateral triangles or if the BS is the corner between two sides of equal length. To have a general case, the author therefore advocates using the mirrored centroid as best estimation.

Figure 18: Moving centroid: case "between"

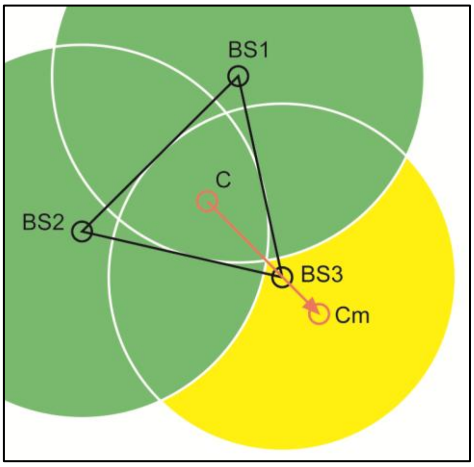


Source: Illustration by author

3.5.2 Case 2: "behind", MS near one BS but not the other two

Figure 19 shows the situation, where the MS is near BS3, but not near BS1 and BS2, resulting in a position in the area filled in yellow. As there is no edge to mirror on, and the shape does not resemble the inside-area of the triangle, the author proposes to move the centroid on the line going through centroid and BS3 (the centre of the circle defining the yellow area). The distance of the movement remains to be defined as does the following movement of the centroid. In the following estimations, the centroid will be moved 1.5 times the distance between itself and the BS in question. Considering geometrical renderings of triangles, this result seems to be near a possible centroid of the yellow shape in question. It is important to note that better results might be found by extensive modelling of the situation.

Figure 19: Moving centroid: case "behind"

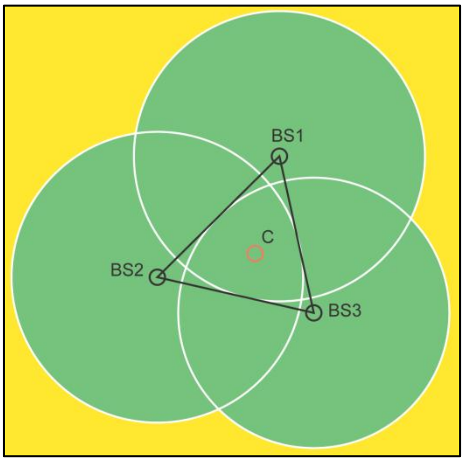


Source: Illustration by author

3.5.3 Case 3: "outside" MS is in none of the defined areas

In the situation where the MS is estimated to be outside the three circles, no improvement of the centroid algorithm can be achieved by means of geometry. The MS could be positioned anywhere in the yellow area indicated in Figure 20.

Figure 20: Moving centroid: case "all outside"



Source: Illustration by author

As no anchor for calculations is available but the RSS-readings to the BS, the only possibility remains in changing the algorithm. The author proposes a simple proximity approach but any other algorithm using no more than the available data could be used as well.

4 Application to real world data

After the analytical discussion, the evaluation of the ideas in a real world setting is approached. To achieve that, a triangulation model has been programmed in Matlab. With help of the application, the algorithms presented can be checked for quality of estimation with real world data. In this chapter, the general idea of the approach will be reviewed, hypotheses will be formulated, the model and the algorithms in use will be presented. Furthermore, the data available and the setting in which the data was obtained will be presented. Later, the performance of the algorithms will be analysed and the results will be discussed. Detailed analysis will help to get an understanding and insight into the mechanisms in question. Later hypotheses will be checked and implications will be formulated.

4.1 Review of the general idea & hypotheses

Not only the strength of the signal between a BS and the MS can be measured, the same principle applies between two BS. These inter BS measurements help to draw a crude "map" of the RSS situation in the evaluated area. This map shall be used to establish a temporal base for an improved centroid approach.

With the ideas presented in the last chapter, four algorithms are programmed, each theoretically more sophisticated than the last. First, as benchmark algorithm proximity will be used. Second, proximity can be improved by using multiple BSs and calculating the centroid thereof. Third, the centroid approach can be improved by moving the estimation towards the BS positions depending on RSS information. Finally, this approach can be improved by accounting for MS positions outside the triangle formed by the BS. This leads to the following hypotheses:

1. Centroid performs better than proximity.
2. Weighted centroid performs better than centroid.
3. Improved weighted centroid performs better than weighted centroid.
4. Inside outside tests improve the performance of the weighted centroid approach in the following order: io3Max > io2.

Given available data, multiple estimates will be averaged over a certain time to calculate the location of the mobile station. For every estimation, the three BS with the strongest RSS will be chosen as the used triangle. By means of an inside/outside-test an initial position for the centroid approach will be obtained. This initial position will be adjusted using RSS-measurements of the MS. The impact of each RSS-measurement on the location estimate is determined by the corresponding inter BS RSS measurements.

4.2 Basic model elements

Different elements are needed to model the real world scenario. These consist of the following objects:

- The **mobile station** to track is situated at position (x,y). The estimation of this position is the goal of the model. To evaluate the accuracy of the approach, the estimated position will be compared to the known position (x,y) from the base data.
- Each **base station** has a fixed position (x,y) and a RSS reading of the mobile station's signal. Furthermore a base station can get the RSS reading of all other base stations (1-k).
- A **central unit** governs the access point information and handles calculation. This results in a matrix of inter BS RSS, a matrix of BS to BS distances and a vector of BS to MS RSS per chosen time unit. If enough readings are available, calculations over time periods (e.g. average) can be used to gain improved results.

The goal was to create a system as flexible as possible within the desired bounds. By means of init files, own configurations can be added containing information about

- the position of the BSs
- data-sources
- algorithms of choice (can be programmed to an available interface)
- size of time-steps of evaluation (changing the time granularity of the model)
- background maps (making the application portable to other sites)

An overview on the system is given in appendix VIII where an UML-Diagram shows further components of the application beside the ones just introduced. These further elements (e.g. a plotter-class for exports) play a minor roll and act as helpers.

It is important to note that the data is analysed retrospectively. This fact can be ignored in all cases but the ones, where inter BS readings are needed. The collection and summation of these adds a certain delay depending on the system speed (how fast can data be measured) and the time step (how much data has to be aggregated). Since inter BS-measurements are used to weight RSSI readings, in theory, the closer in time these are together, the better.

4.3 Algorithm overview

This chapter is divided in two parts. In the first part, the author describes the general operation of the application and hence the triangulation model. The second part is dedicated to the different algorithms. The division into two parts is motivated by the application accommodating different algorithms for estimation of the MS position.

BS and MS are known from the previous chapters, estimation steps stand for the "time-bins" of aggregated information that can be evaluated. These bins have a start- and an end-time. All measurements between MS and BS falling in between are aggregated to have one relevant value per time-bin (e.g. taking the max, min, average). The size of these time-bins can be varied, the author settled with a 3 second interval to account for the moving speed of a person (holding the MS or blocking the signal).

4.3.1 General model

The following stages give an insight in the workings of the application. The stages are modelled for real-time measurements. In the model used for triangulation, stages 1 to 3 where combined due to the fact that the data is analysed retrospectively. For details see the following chapters.

Stage 0: initialization

In stage 0, the whole system is created. BS are set up and registered with the central unit, necessary flags for calculation (bin sizes and the like) are created.

Stage 1: Surroundings

Every BS measures RSSI to every other BS

Stage 2: Selection

Every BS measures RSSI to the MS

The three BSs with the strongest RSSI will be the nodes used in triangulation

Stage 3: Triangulation

Here, the estimated position of the MS is calculated using the different algorithms. Details will be given in Chapter 4.3.5. The general approach is to calculate the centroid-point of the triangle and apply the chosen weighting strategy.

Stage 4: Presentation of results

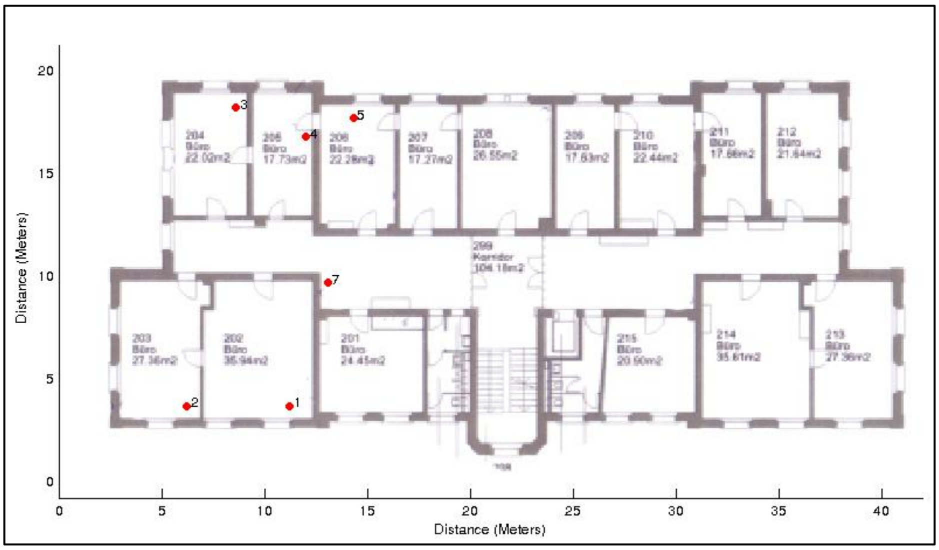
The position of the mobile station is then presented and estimated against a known position for evaluation purposes. This step takes place after all estimations have been made, but could as well be done in real time.

4.3.2 *Stage 0: Site, base stations, measurement positions*

Data collection took place inside the University-Building of the IAM at Engehalde (Neubrückstrasse 10). Figure 21 shows the layout of the floor, where the BS where deployed. The setting features multiple walls that are rather thick, given the historic building.

BS where positioned in the left wing part. As BSs six "Gumsticks Overo Fire" where used. Figure 21 shows the BS named {1,2,3,4,5,7} and their position on a meter-scale with arbitrary zero point. For details see appendix IV.

Figure 21: Base-Station positions

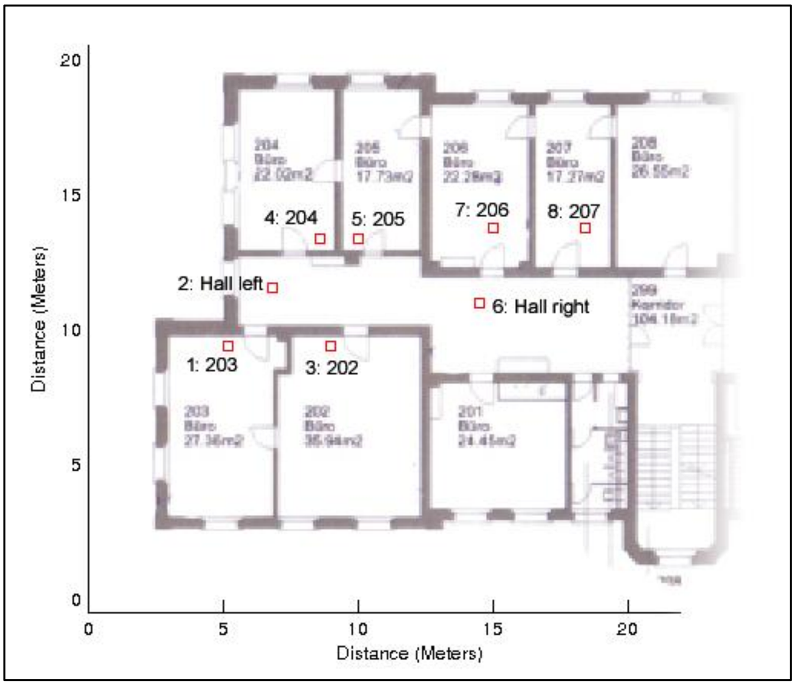


Source: University, added information by author

The Data was gathered on the 11 July 2012 in the context of the “Location Based Analyser” Eurostars project E!5533. The test data was obtained with a "HTC Wildfire" MS.

During collection, the MS was positioned in 8 different places on the left-wing part of the building. The positions are shown in Figure 22. The numbers indicate the chronological order in which the evaluations took place and the identity of the evaluation position. For details see appendix V.

Figure 22: Evaluation positions



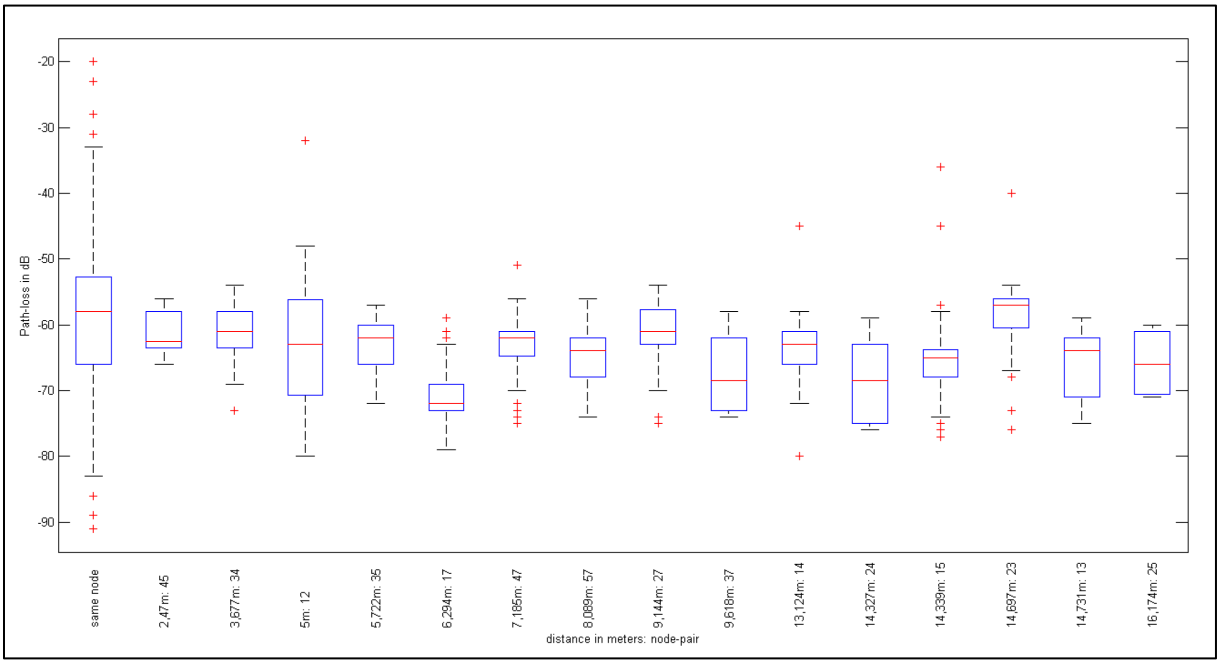
Source: University, added information by author

4.3.3 *Stage 1: Inter BS RSS data*

At the same time as the MS-measurements took place, inter BS RSS data was gathered too. The data file obtained contains 891 RSS entries that can be used in localisation. Of these 233 (26%) where readings where the sender and receiver are the same BS (due to reflection). 658 readings could hence be used to calculate the inter BS weighting table.

Due to the different distance between two stations, the range of the RSS received varied greatly. Figure 23 gives an overview of the distribution of RSS-readings per node-pair and associated distance sorted from shortest distance to greatest (e.g. for the second column: 2.47m stands for the distance in meters, 45 for the node-pair in question, nodes 4,5).

Figure 23: Box plot: Path loss per node-pair and distance

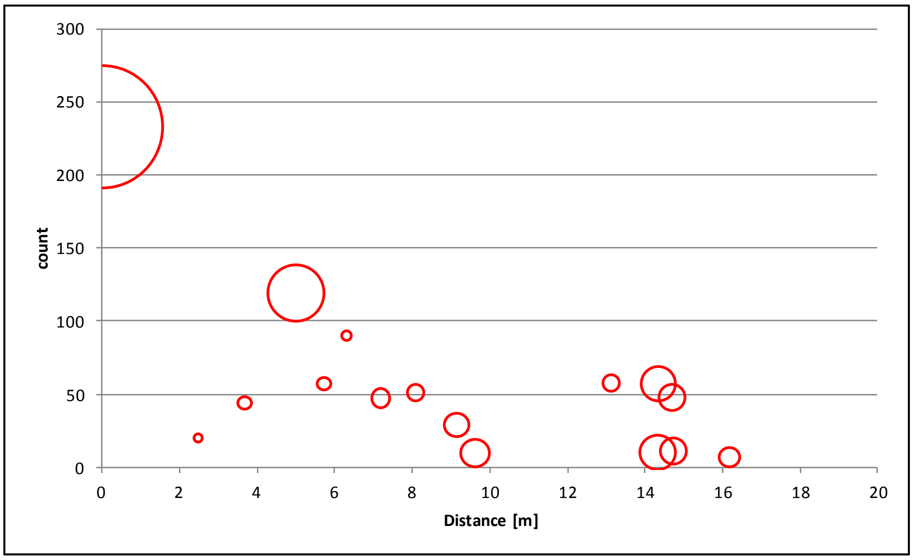


Source: Inter BS Data filtered from base file

There is not a great change in the average readings between the nodes depending on the distance. Certain node-pairs however tend to have greater variance between the readings than others. Greater distance and greater count both are linked to greater variance as can be seen in Figure 24, where the axes stand for count and distance while the diameter of the bubbles indicates the variance of the readings available. However in all cases with overall greater variance, BS1 and BS2 are a part of the pair in question as could be observed in Figure 23. This appears to be due to the fact that the two nodes are on one side of the floor, while the other nodes (safe 7) are on the opposite side (see Figure 21 for positions). For details on variance see appendix VI.

It must be pointed out that the count of inter BS readings varies greatly. While the link between BS1 and BS2 has 119 readings, the one between BS2 and BS5 has only 7. While there are exceptions, generally a greater distance seems to yield less readings as Figure 24 shows. This can be attributed to the fact that attenuation of signals or signal loss due to multipath and the like plays a greater role, when the distance gets bigger.

Figure 24: Distance / count of readings / variance



Source: Inter BS Data filtered from base file

Given a six BS deployment, 15 readings between different pairs are necessary to create an entire net of inter BS readings. One possible optimisation is to reduce the needed inter BS readings only to the BS that actually participate in the centroid construction. This optimisation, however, becomes difficult, if the MS is mobile thus causing changes in the set of BSs forming the triangle. Reducing the number of readings for the inter BS net to only the necessary ones would allow for a more efficient inter BS net building as less readings would have to be collected. Collecting readings to build a complete inter BS net might lead to a great time distance between the BS to MS information and the inter BS readings as some readings might be harder to obtain (e.g. secluded nodes).

To get a whole inter BS net, the author had to decide to set the size of the inter BS time bands to 5 minutes. This is far from optimal to account for the movement of people but at least helps to account for walls, furniture and other rather immobile factors of attenuation. As more readings needed to form a denser net equals more overhead and interference on the Wi-Fi-band, depending on the situation longer time-bands might be an acceptable trade-off.

4.3.4 Stage 2: MS to BS RSS data

The data available from the 11 July 2012 starts at 08:47:52 and ends at 09:34:31. It consists of 404 entries with timestamp, BS-number and RSS-reading in dBm. Of these 132 (33%) did not fall into a time band with a known evaluation position (e.g. data before/after the evaluation took place). The data without known positions fall into the time before 08:48:00 and after 09:31:00, therefore closing the gap between the different estimation periods does not yield further relevant readings where a position to evaluate against could at least be approximated. Table 2 gives an overview of the data. Here readings are split by the BS receiving the signal. More detailed overview of the RSS-readings per time frame and BS is given on appendix VII.

Table 2: Readings per BS and time frame

Location	Time Frame	BS 1	BS 2	BS 3	BS 4	BS 5	BS 7	Sum
203	TF1	10	13	14	12	10	11	70
hall left	TF2	2	2	2	1	1	2	10
202	TF3	12	12	7	10	9	12	62
204	TF4	2	3	2	1	2	5	15
205	TF5	2	1	2	5	1	3	14
hall right	TF6	4	2	3	3	3	5	20
206	TF7	3	3	2	8	11	6	33
207	TF8	1	8	7	9	11	12	48
NA	NA	13	21	17	25	26	30	132

In Chapter 3.1.1 the goal for the size of evaluation time-bands has been defined as being between three to five seconds. As the different triangulation (safe proximity) need at least 3 readings, only time bands with readings from three or more BSs can be used in triangulation. This further reduces the available Data for triangulation in certain time bands. For example it is highly unlikely that the Data received in time frame 2 spread over 4 Minutes yield usable information for triangulation.

4.3.5 Stage 3: Triangulation

Overall four algorithms have been implemented and evaluated to answer the hypotheses formulated in the last chapter. The following subchapters give an overview of the algorithms.

Proximity

The first algorithm implemented is the proximity algorithm, due to its simplicity it gives a good base to build on and to evaluate against.

In every estimation step, the algorithm checks the RSSI between MS and all the BS, the BS with the strongest signal is selected, its position returned. Should two or more BS have exactly the same signal strength, the position of the one first added to the system is returned.

Centroid

In every estimation step, the algorithm checks the RSSI between MS and all the BS. The three BS with the strongest signal are selected. Again if two or more BS compete for position three by having exactly the same signal strength, the first one added to the system is selected. The estimation returned is calculated by the basic non-weighted formula given in Chapter 2.3.4.. Time slices, where less than tree BS report a signal are omitted from calculation.

Basic weighted centroid (wlcentroid)

A version of the centroid algorithm improved with weighting.

The centroid of the BS triangle is taken as starting point. Now vectors to all three nodes are calculated and weighted by the RSS converted to distance, with the formula introduced in Chapter 2.3.4 under weighted approach. This basically results in moving the estimation towards nodes with stronger signals.

Improved weighted centroid with inside/outside test (bwcentroid)

A more complex version of the centroid algorithm where the RSSI weightings are further adjusted with inter BS weights.

Inter BS weights are calculated first. The position of the BS is known, the central unit calculates the distances between all the BSs in the initialization phase (*ds*). With the available inter BS Data, a matrix of inter BS Data readings for multiple time-slots is established[14].

In every estimation step the algorithm checks if inter BS data for the relevant nodes exist. If no inter BS data is available, the MS data is not evaluated. Else, using a simplified version of the log-distance path loss formula introduced in Chapter 2.2.5, the known inter BS distance is converted into the theoretical free space path loss (*fspl*) by using a previously measured $L(d_0)$ at 1 meter of -45 dB and a path loss exponent set to 2 (free space propagation). The fraction of the measured inter BS RSSI to fspl hence allows for the estimation of the attenuation of the signal compared to free space propagation and can be used in weighting the RSS readings from MS to BS. This accounts for the situation, where the RSSI reading from MS to BS might be weaker due to signal weakening obstructions[15].

Furthermore a test is applied, where the initial position (the centroid) is changed, if the MS position is estimated to be outside of the triangle. This estimation is the inside/outside-Test presented in Chapter 3.3. bwcentroid will be run with two different inside/outside algorithms to evaluate the different performances of the approaches already discussed in theory. These tests are called

- **io2** (distance calculation: triangle height)
- **io3max** (signal strength stronger signal as threshold)

4.4 Model output

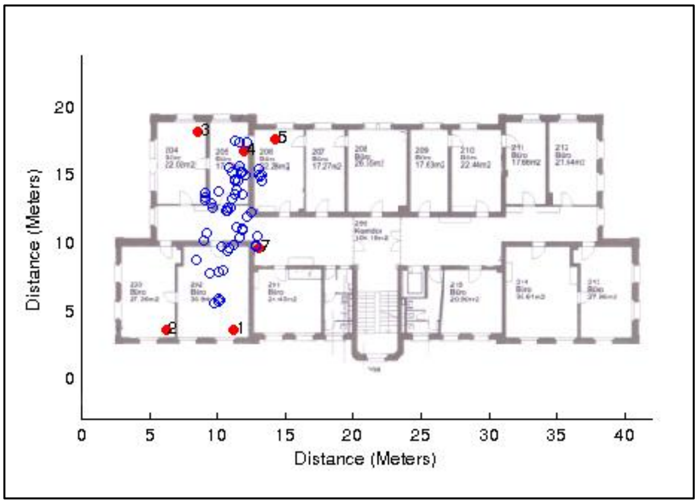
The Matlab model prints out three different charts for cockpit-view. An example output of the first chart can be seen in Figure 25. Here estimations are placed on the defined map together with the BS. This helps getting an overview of the scene. It is also the view that would most

[14] This is possible, as past data is evaluated. When doing real time measurements and weighting this would lead to a minor delay. Depending on the Hardware used (and the frequency of inter BS measurements) this step is critical.

[15] It is not however possible to be sure if the obstruction is only between the BS or also between BS and MS.

likely be the final result of a triangulation-algorithm in a productive environment as only estimations are shown. In Figure 25 red dots show the known positions of the BS, while blue circles stand for estimations. Figure 25 shows example data collected over the duration of one hour on multiple evaluation positions.

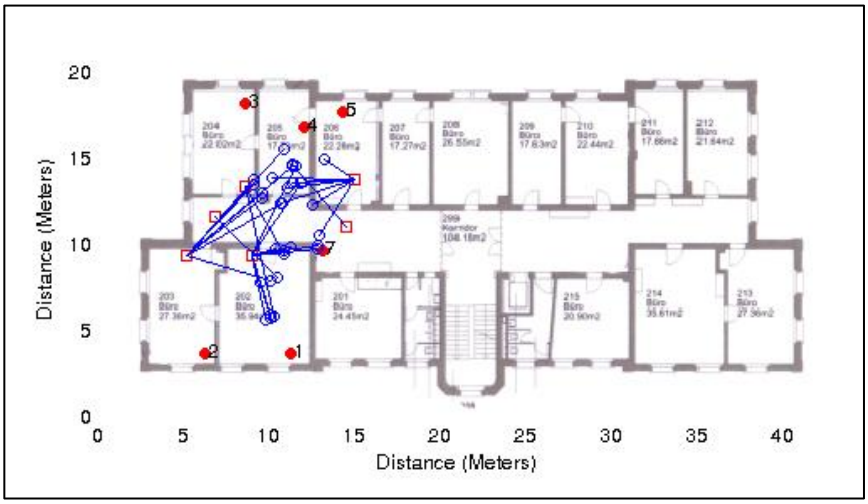
Figure 25: Estimation view



Source: Matlab, triangulation model

The second view helps in assessing the performance of the algorithms checked. Here not only estimations are shown (as displayed in Figure 25) but these are also linked with the known evaluation positions (e.g. the positions, where the MS was placed during data collection). This figure uses the same example for the measurements over an hour therefore indicating multiple real-estimated position pairs. For some estimated positions there is no known real one (evaluation position) because the device was moving between positions, or the data received is from before/after the evaluation period. Therefore, the such estimated positions are not shown in this second cockpit view Figure 26 shows an example output for the evaluation view. Again red dots show the known positions of the BS, blue circles stand for estimations. The red squares are the known evaluation-positions. Blue lines connect the estimations with the corresponding evaluation-positions.

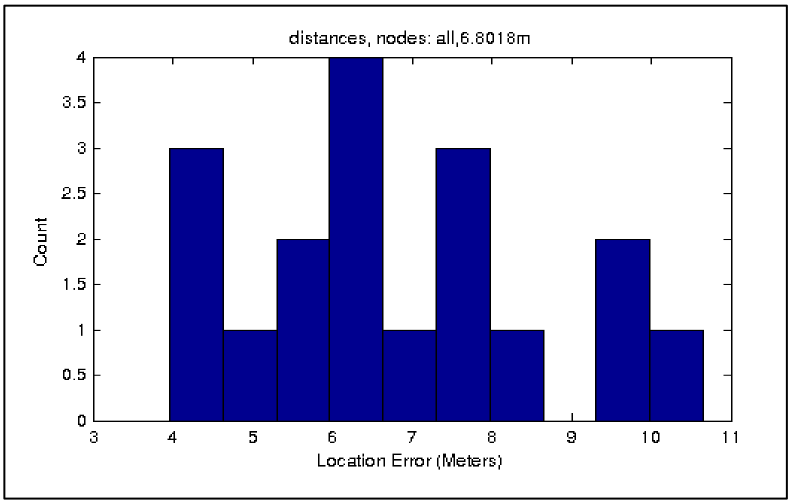
Figure 26: Evaluation view



Source: Matlab triangulation model

The third view in Figure 27 is used to compare different approaches against each other. It contains a histogram over the location error in meters (the distances between evaluation and estimation; the length of the blue lines in Figure 26). The header also contains the average over the estimations (in Figure 27, the average is 6.8018 meters).

Figure 27: Histogram view



Source: Matlab triangulation model

It is furthermore possible to export the calculated data as txt-file in form of a "mastertable" containing for all the time-bands the signal strength from all BSs, chosen BSs for triangulation, corresponding estimation position, real position and the calculated location error between the two. This can be used to store and compare different algorithms and settings outside of Matlab if desired.

4.5 Results & discussion

4.5.1 Overall results

Since the different algorithms have different requirements regarding the MS to BS readings available and the availability of inter BS measurements, simpler algorithms produce more results as more time bands have valid data. Table 3 gives an overview of the count of results of the different algorithms with the available data. The first column of data is the number of time-bands that could be used in estimation (with enough BSs available to run the algorithms). In the second column of data, only the estimations with a known evaluation position are counted[16]. Therefore, the number of estimations with evaluation is always smaller than the number of estimations. Proximity needs only one BS, centroid and wlcentroid use three BSs, and the algorithms with an inside/outside test further need available inter BS measurements. Hence, the more complex the algorithm gets, the lower the number of estimations.

Table 3: Count of results per algorithm

Algorithm	N of estimations	N of estimations with evaluation
proximity	109	68
centroid	56	31
wlcentroid	56	31
bwcentroid(io2)	22	18
bwcentroid(io3Max)	22	18

Source: model

Depending of the quality of the network scanner and the density of the collected measurements, it can happen that a certain time-bin lacks RSS-measurements of one or two BSs, while three are needed for triangulation. To allow for fair comparison, only evaluation positions for which all algorithms can be run will be compared.

As the goal in estimation is a precise localisation of the MS, the distance between real and estimated position (localisation error) will be used in judging the precision of the different algorithms. As a perfect estimation would have 0 localisation error, a smaller localisation error signifies a better estimation. Table 4 gives an overview of the performance of the different algorithms. Values indicated are the smallest error (Min), the biggest error (Max) and the average error (Avg):

[16] These are estimations are from before or after the evaluation phase, or where the person evaluating was moving from one evaluation position to another (breaks).

Table 4: Distance per Simulation

Algorithm	Min	Max	Avg
proximity	3,962	9,434	6,401
centroid	1,555	8,164	4,898
wlcentroid	2,206	9,530	5,626
bwcentroid(io2)	3,962	10,301	6,891
bwcentroid(io3Max)	3,962	10,301	6,795

Source: model

With the given situation, the results in Table 4 imply that using a centroid-algorithm over the proximity approach improves the quality of the estimation. Weighting the centroid (wlcentroid) does not however yield further improvement over the centroid approach. Improving wlcentroid with an inside/outside test and the corresponding changes to the estimation base (moving the centroid outside the triangle) yields no better result than the proximity approach. This can be seen in the fact that the maximum location error and the average are even worse with both bwcentroid algorithms than with using proximity.

To understand the reasons for this behaviour, particularly the performance of bwcentroid, more detailed information than the overview data in Table 4 is given in the following chapter.

4.5.2 Focus on the bwcentroid algorithms

Table 5 gives an overview of the performance of the different algorithms at the different estimation positions. The first column marks the different estimations by starting time of the time-band. In the following columns the localisation error of each algorithm[17] is indicated. This allows for an in depth comparison of the algorithms.

Table 5: Estimation per algorithm (loc. error [m])

Time	proximity	centroid	wlcentroid	bwcentroid (io2)	bwcentroid (io3Max)	bwcentroid (io3Avg)	bwcentroid (io3Min)
08:48:12	5,787	5,890	5,388	7,772	5,787	7,772	7,772
08:48:18	9,434	8,164	9,434	9,434	9,434	10,656	10,656
08:48:27	5,787	5,890	2,206	5,787	5,787	5,787	7,772
08:48:33	7,906	8,164	7,848	7,906	7,906	7,906	7,906
08:48:39	9,434	5,117	9,530	9,434	9,434	9,434	6,479
08:48:48	5,787	6,546	5,216	10,301	10,301	7,105	7,109
08:48:54	9,434	5,117	9,434	9,434	9,434	9,434	6,479
08:49:03	5,787	5,465	5,787	8,120	8,120	8,120	2,899
08:53:39	7,923	4,964	7,284	7,923	7,923	7,923	7,923
08:58:42	4,111	3,986	4,105	4,111	4,111	4,217	4,728
08:58:45	4,111	3,986	3,967	4,111	4,111	4,111	4,217

[17] Including two further algorithms modelled for evaluation purposes bwcentroid io3Avg and io3Min, for details see Chapter 3.4

Time	proximity	centroid	wlcentroid	bwcentroid (io2)	bwcentroid (io3Max)	bwcentroid (io3Avg)	bwcentroid (io3Min)
09:01:39	6,351	1,555	6,335	6,351	6,351	6,351	6,351
09:02:51	6,351	1,581	3,369	6,351	6,351	6,351	6,351
09:02:54	6,351	3,880	2,779	6,351	6,351	6,351	6,351
09:03:27	6,351	3,880	4,004	6,351	6,351	6,351	6,351
09:04:15	5,826	7,858	6,257	5,826	5,826	5,826	5,826
09:26:36	4,519	4,042	4,414	4,519	4,779	4,779	4,779
09:26:42	3,962	2,087	3,911	3,962	3,962	3,962	3,962

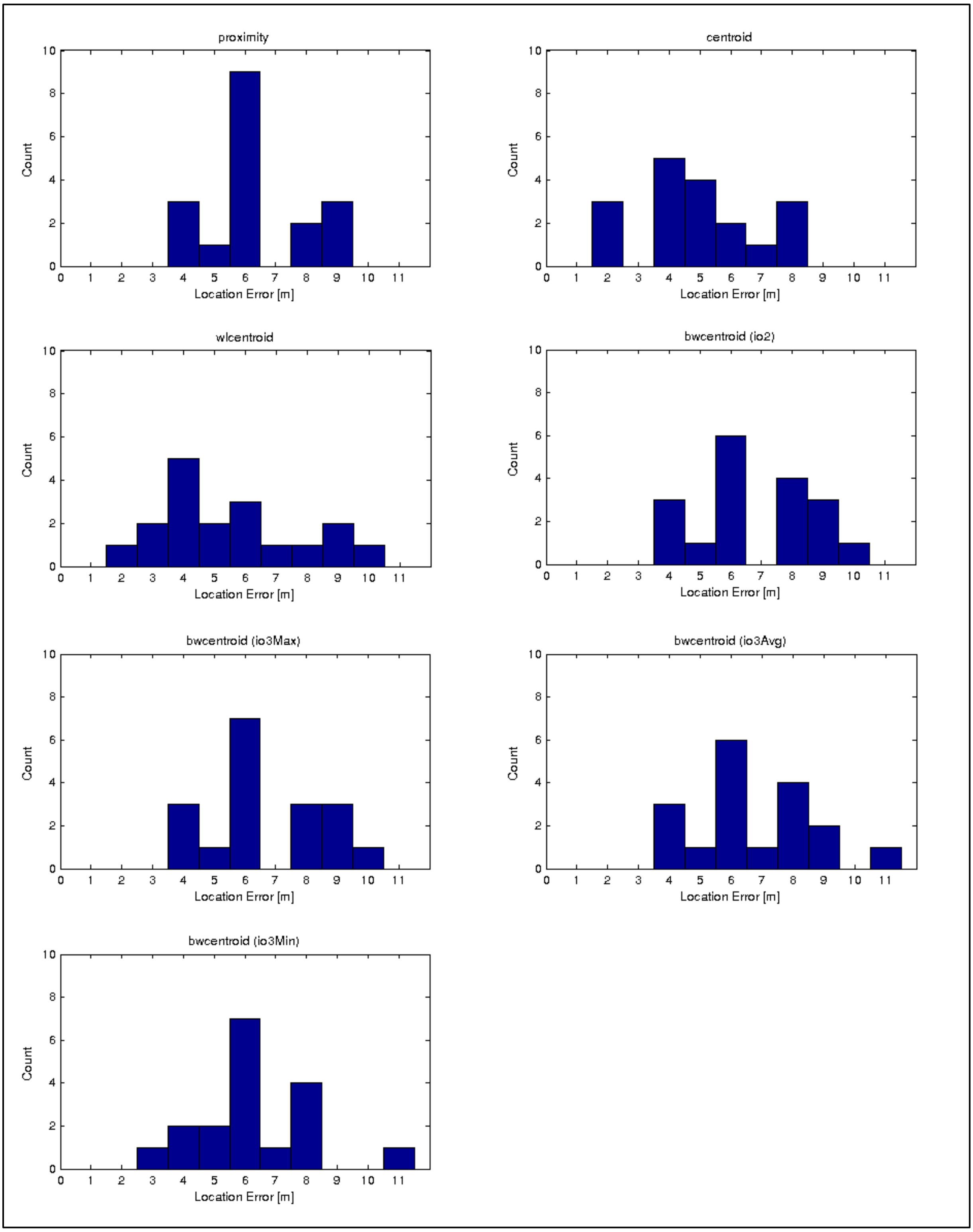
Source: model

The data shows that the bwcentroid algorithms tend to resort to proximity in all but three cases for io2 and three cases for io3Max (For bwcentroid io2, these are the estimations with timestamp 08:48:12, 08:48:48, 08:49:03 and for bwcentroid io3Max, these are 08:48:48, 48:49:03 and 09:26:36). This is confirmed by the same result for proximity and the bwcentroid algorithms. Recall that this is only done, if the inside/outside test results in the estimated position being outside of all three threshold circles defined by the inter BS measurements. Hence, the observed evaluation of the bwcentroid's performance is very much influenced by the geometry of the deployment.
Furthermore, the remaining estimations, where the MS is inside at least one threshold circle, have a worse performance than the proximity approach as can be seen in the histograms depicted in Figure 28. This factors together lead to the bwcentroid algorithms having the biggest average localisation error of the algorithms discussed.

The performance of the inside/outside test explains part of the problems encountered with the bwcentroid algorithms. As most of the positions are evaluated as being “outside”, the inside/outside-tests used (while delivering good results in simulation) might be too strict in a real world scenario with fluctuating RSS.

Hence, the bwcentroid algorithm with the io3Avg and the io3Min inside/outside test were included in Table 5. Still even with the less strict tests, the results did not improve. While averaging the inter BS readings accounts for complex triangle geometry (e.g. pointy triangles, flat triangles) by generating a bigger inside area, the minimum test (accepting the weakest signal as the threshold for inside) in the simulation always resulted in an area bigger than the triangle (and was therefore rejected). Both algorithms have been tested and even with these very forgiving approaches, no "inside" position could be found and still most of the positions evaluated are evaluated as "outside all thresholds".

Figure 28: Histograms of results



Source: model

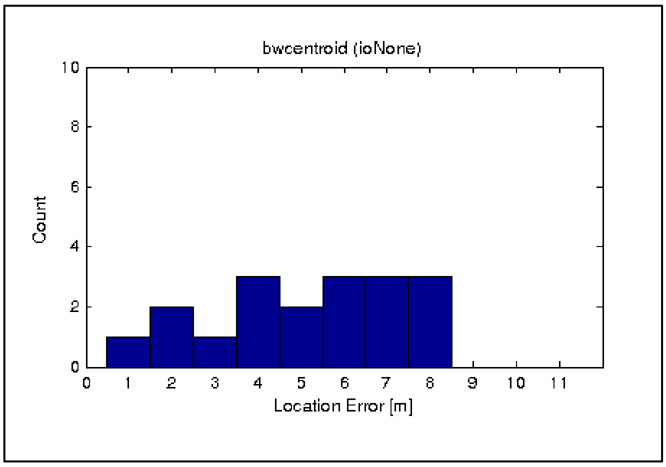
In Order to test the impact of inter BS weighting, the author decided to isolate the inside/outside test from the bwcentroid algorithm (e.g. adding a test that always responds with “inside the triangle”, *ionone*). This yields significant better results as shown in Table 6 (an update to Table 4) and Figure 29:

Table 6: Bwcentroid additional approaches

Algorithm	Min	Max	Avg
Proximity	3,962	9,434	6,401
Centroid	1,555	8,164	4,898
Wlcentroid	2,206	9,530	5,626
bwcentroid (io2)	3,962	10,301	6,891
bwcentroid (io3Max)	3,962	10,301	6,795
bwcentroid (io3Avg)	3,962	10,656	6,802
bwcentroid (io3Min)	2,899	10,656	6,328
bwcentroid (ionone)	1,106	8,293	5,142

Source: model

Figure 29: Bwcentroid without Inside/Outside-Test



Source: model

In summary, adjusting the weighted centroid with inter BS weights is an improvement to the weighted centroid. The inside/outside-test however did not improve the results.

4.5.3 Focus on inside/outside-test

To understand the failure of the inside/outside test, the evaluation positions as well as the different nodes and triangles playing a role in the cases estimated will be further examined.

Table 7 shows the number of time-bands with readings (RSS from at least one node available) and number of estimates per evaluation position. The evaluation positions are sorted by time of evaluation (for a location map see Figure 30).

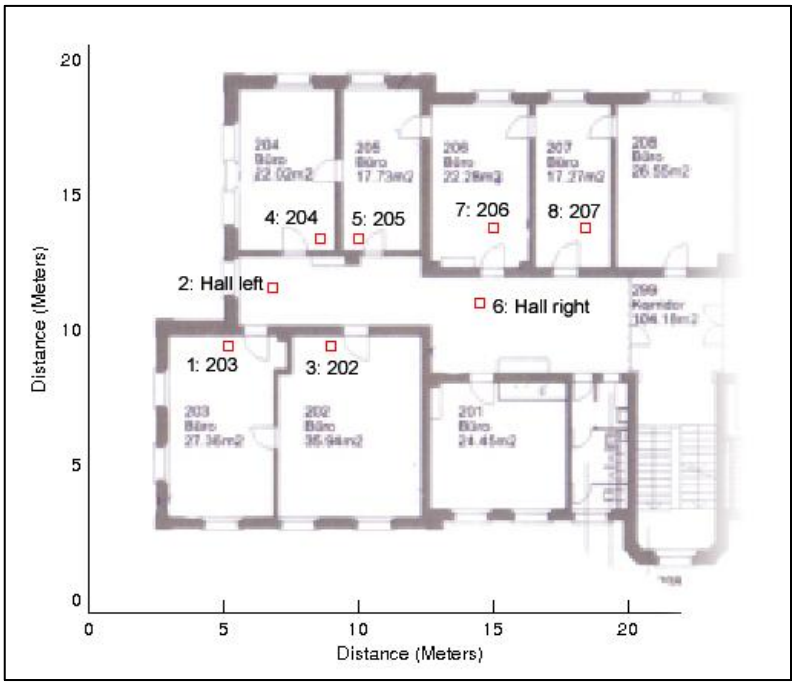
Table 7: Estimations per location

Evaluation position	N of readings	N of estimations
203	16	8
hl	7	1
202	14	6
204	8	1
205	-	-
hr	4	-
206	19	2
207	-	-

Source: model

The non uniform distribution of the cases is obvious. The estimations appear mostly on the left side of the area where the data was gathered. For some positions (205 and 207), no triangulation data was available at all, for position hall right, the readings did not include enough BS to estimate a BS position.

Figure 30: Evaluation positions



Source: University, added information by author

Table 8 shows the usage of the nodes, where node stands for the number of the node in question. The table shows the count of usage of the nodes in triangulation with the corresponding count of having the strongest to weakest RSS readings. As the three nodes with the strongest signal are used in evaluation, top, second and third highest RSS encountered are individually counted per node. As all nodes are treated equally in triangulation and one node can only be either top, second or third, the sum of occurrence of each node can be used in an assessment.

Table 8: Usage of nodes in estimation

Node	N times top RSS	N times second RSS	N times third RSS	Sum of occurrence	% of occurrence
1	0	4	5	9	16,7%
2	9	0	3	12	22,2%
3	3	2	1	6	11,1%
4	0	5	3	8	14,8%
5	1	4	4	9	16,7%
7	5	3	2	10	18,5%

Source: model

Table 8 shows that BS 2 and BS 7 are used most frequently in triangulation. While BS 7 is the centre node in the hall (and can thus be considered a candidate for most of the evaluation positions), BS 2 is situated at the lower left corner (see Figure 21). The top-position of BS 2 does not however surprise, as BS 2 is also a suitable candidate given the fact that most of the positions evaluate are situated on the left side of the area used in modelling.

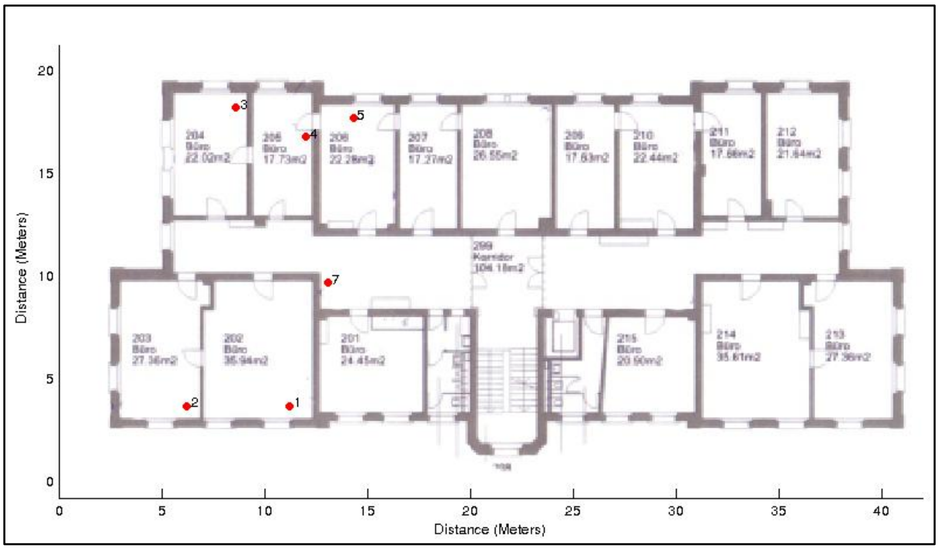
Table 9 shows the triangles estimated (e.g. 124 is the triangle formed by BS 1, 2, 4 - for node positions see Figure 31) and how often each triangle appeared in estimation. The performance of the different triangulation algorithms on the respective triangles can be seen in the corresponding rows. Table 9 shows that the most precise measurements tend to be made in triangle 457 and 157 - triangles at the right border of the area of evaluation.

Table 9: Triangles estimated

	124	125	127	157	234	235	245	247	347	457
triangle count	1	2	3	3	2	2	1	1	2	1
proximity [m]	6,35	6,85	6,18	4,25	9,43	5,79	5,79	6,35	8,67	3,96
centroid [m]	1,55	5,21	5,21	4,00	5,12	5,89	6,55	1,58	8,16	2,09
wlcentroid [m]	6,33	6,54	4,35	4,16	9,48	3,80	5,22	3,37	8,64	3,91
bwcentroid (io2) [m]	6,35	8,02	6,18	4,25	9,43	6,78	10,30	6,35	8,67	3,96
bwcentroid (io3Avg) [m]	6,35	8,02	6,18	4,37	9,43	6,78	7,11	6,35	9,28	3,96
bwcentroid (io3Max) [m]	6,35	8,02	6,18	4,33	9,43	5,79	10,30	6,35	8,67	3,96
bwcentroid (io3Min) [m]	6,35	5,41	6,18	4,57	6,48	7,77	7,11	6,35	9,28	3,96
bwcentroid (ionone) [m]	1,11	5,28	4,90	4,36	6,06	6,75	7,11	1,82	8,07	2,40
avg per triangle [m]	5,09	6,67	5,67	4,29	8,11	6,17	7,43	4,82	8,68	3,53

Source: model

Figure 31: Base-Station positions



Source: University, added information by author

A notable triangle is 235 with great distances between the nodes and a BS inside the triangle. Situations, where one of the corner BS's, could not be replaced by the inside BS (BS 4) should be rare in theory. This can happen, if readings from different BS are not available or not numerous enough to average or account for outliers.

4.5.4 Focus on quality of inter BS measurements

Another possible source of the shortcomings of the inside/outside test might be the inter BS measurements. These are in all cases used as a reference base for the calculations. As stated in Chapter 4.3.3 the inter BS data is not dense enough to create a net with narrow time bands. The best case would be equal sized time bands in inter BS data and BS to MS data (e.g. three seconds). In reality the data did not allow for shorter time bands than five minutes with the inter BS data. This comes short of providing dynamic updates to compensate for fast fading effects and can only help to isolate the impact of semi-dynamic propagation changes. If for a link, multiple readings where available, the strongest was taken into consideration. It was not possible to collect additional data, as the Wi-Fi testbed was updated and is no more collecting inter BS measurements.

This leads to the situation where MS to BS measurements are compared against a collection of inter BS data points spread over time not stemming from the same situation (e.g. people blocking the nodes, signal only received if a door was open...). Hence during runtime, situations can occur, where the relationship between distance and RSS on record varies greatly making the usage of the inside/outside tests less accurate. Again, more RSS readings would allow for averaging and accounting for outliers.

4.5.5 Further possible sources of deviation

While the last subchapters put the focus on different part of the algorithms to analyse the performance, this subchapter will list further possible sources of error.

During the analytical discussion, the author was convinced that the outside position (e.g. all three BS report the MS to be outside the thresholds) would be the exception. Hence, only small effort went into the estimation of the positions. Since these are approximate only and results are not weighted by RSS (as the base for moving the centroid is missing), outside positions expectedly have a greater error margin.

As no controlled measurements such as LOS versus NLOS were available for those particular deployments, the systems could not be checked in a controlled environment. Controlling the environment cannot be done by modelling datasets as these datasets would mostly be modelled exactly under the same assumptions used in the model (e.g. propagation formulas).

A further possible source of deviation is found in the exactness of the placement of BS and evaluation positions. As the position of the BS is the base for all calculations in triangulation, moving a BS in the model leads to an error that cannot be eliminated once established. The same thing applies to the evaluation positions the estimations are compared against. Since modelling took part in an indoor environment, placement could not be done with line of sight (triangulation) or positioning aided by GPS. The positions had to be approximately measured and marked on a map. This could introduce an error on the layout but the expected effect on the localisation error should not be significant.

4.6 Hypotheses check & implication

In Chapter 4.1 the following hypotheses have been formulated:

1. Centroid performs better than proximity
2. Weighted centroid performs better than centroid
3. Improved weighted centroid performs better than weighted centroid
4. Inside outside tests improve performance of the weighted centroid approach in the following order: io3Max > io2

As already mentioned in the previous chapters, the data available is rather sparse. This should not be forgotten while checking the hypotheses and reading the following thoughts.

As shown in Chapter 4.5.1., the modelling shows that the centroid performs better than the proximity-approach. Hence the corresponding null hypothesis can be rejected.

Weighting the centroid with the MS to BS RSS data did not further improve the result. This is most probably due to the fact that the inter BS RSS data was to sparse to allow for compensation of fast fading. Also, the improved weighted centroid approach as introduced in Chapter 3 did not improve the estimation of the MS's position. It cannot be said if io3Max > io2 as the two did only differ in two out of 18 estimations. Here io3Max gives a slightly better estimate than io2, however there is not enough data to reject the null hypothesis.

The outcome of weighting the centroid with inter BS measurements without moving the centre of estimations (e.g. the centroid) was not formulated as a hypothesis. During the analysis this question was answered too. Weighting the centroid with MS to BS readings can be improved, if these readings are weighted themselves with inter BS measurements gained from cross monitoring the available BS's. The improvement can be made even if the inter BS data is rather sparse and has to be aggregated over time.

Hence the author concludes that this form of inter BS weighting (aggregated over longer periods of time) helps in accounting for walls and furniture thus improving the estimation. Since the model chooses the strongest signal available for calculations, low attenuation situations are preferred if more than one reading is available. This form of inter BS data is therefore not sufficient to account for temporal anomalies due to moving people or changing factors (open/closed doors and the like). It is important to note that this assumption can only be verified with further experiments and data.

The model presented allows for the estimation of a mobile station to an average error of 4.9 to 6.9 meters (see Table 6) depending on the algorithms used and given the already introduced sparseness of the data. A known error in estimation with inter BS weighting (even if by a different approach) is the one presented by Kim et al. (2011). Using trilateration with RSS adjusted with inter BS measurements, the authors could reduce the localisation error indoors down to 2 meters (Kim et al. 2011, p. 236). It remains to be seen, if the presented approach can yield a comparable precision.

5 Summary & outlook

The author gave an overview of the technical terms and concepts of estimating positions by triangulation in Chapter 2. In Chapter 3 the weighted centroid approach was developed. It includes the improvement of the simple weighted centroid algorithm by means of inter BS measurements gained from cross monitoring BS's. Situations on the triangle where discussed. With the insights gained in discussion, tests to assess whether the position of a MS was inside or outside a triangle of BS's were developed and modelled in Matlab. In Chapter 4, hypotheses where formulated and a triangulation model and different triangulation algorithms were programmed in Matlab. A dataset of Wi-Fi measurements on the RSS provided by the university was presented and analysed. With the triangulation model and data, the algorithms where ran and results could be compared. While it could be shown that weighting the centroid with RSS adjusted by inter BS readings improved the result of the centroid algorithm, the inside/outside tests did not perform as expected, to a certain extend due to not having enough granularity in the data and mainly situations outside the triangle formed by the BSs.

As this leaves room for improvement, the author proposes the following steps for further study. To make sure calculations are correct, the algorithm should be checked in a controlled environment with different settings similar to the situations on the triangle discussed in Chapter 3.2. Another approach leading to similar conclusions would be using the algorithm with different datasets stemming from different situations and buildings having different density and granularity.

Assessing different situations with the given algorithm is one step, but the author is convinced that the algorithms used can be improved further. Discussing the inside/outside tests, results suggested that different triangle geometries favour different inside/outside algorithms. Delving deeper into inside/outside tests and constructing multistate-models choosing the appropriate algorithm for a given triangle could greatly improve the results of the test and hence the performance of the algorithm.

Since most of the positions where estimated outside the given thresholds a better estimation of the outside position as new centroid base is a must. This is a point, where the algorithms lose precision. In a similar manner, weighting the outside positions would expectedly yield better results. Therefore vectors for weighting covering the areas in question would have to be established.

An improved model could furthermore look into BS specific metrics. Hence measurements of BS's with greater variance in RSS could be adjusted thus yielding better bases for estimation. This might however make gathering of further datasets necessary thus contradicting the idea of a system as simple as possible.

As for now, the Matlab model is not calculating the positions in simulated real time but retrospectively assessing available data. Changing this will not allow for fixed inter BS bin sizes anymore (as the granularity and density of the data will not be known) and will therefore add further challenges. This would need the dynamic building of inter BS nets. This could be improved by assessing what data is missing and calculating the probability of arrival of said data (e.g. by means of Poisson distributions). This might allow for smaller but dynamically sized inter BS net time bins, even if the granularity of the data is not changed.

A further field of study is the situation where granularity and/or density of data is low. Available data could be enriched with past data, some data could even be gained by simulation (again contradicting the simple system). This is even more important when the MS in question is moving and not fix as in the situation presented.

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Appendix

App I. Indoor applications and technology applied

Indoor tracking can be achieved using different transmitter bases.

Kolodziej et al. (2006, p. 226f) list the following technologies employed in indoor tracking:

Product	Transmitter & Signal	Position Method
MIT Cricket	Beacon (418-MHz + Ultrasound)	TOF Lateration & Proximity (with one beacon)
Ekahau Positioning System	Existing Wi-Fi (802.11)	Location fingerprinting using signal strength
Microsoft Research RADAR	Wi-Fi (802.11)	Location fingerprinting & triangulation (lateration)
AeroScout	Active RFID tags or Wi-Fi devices (802.11)	TOF triangulation (TDOA for absolute location; RSSI for symbolic location)
BLIP Systems BlipNet	Bluetooth (Mobile Device)	Inquiries and paging

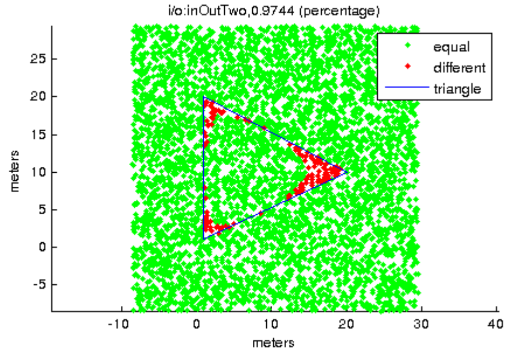
For further applications see: Hightower (Hightower et al. (2001), p. 61)

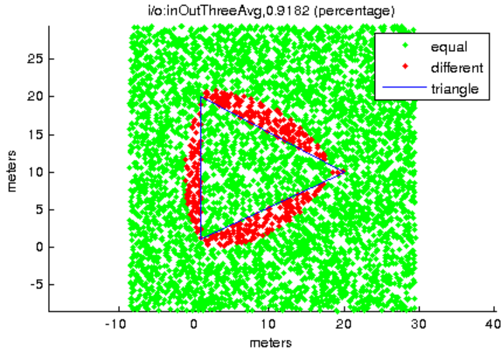
App II. Situations on the triangle

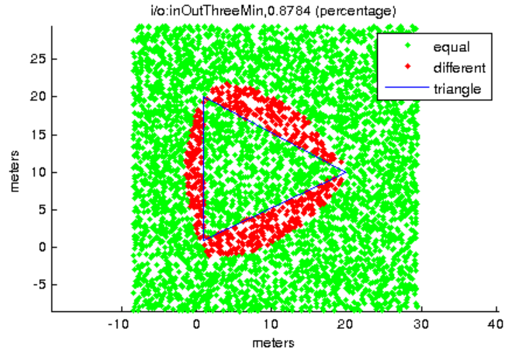
N	Description	Signal strength	Similar to Cases	MS position
1	LOS/NLOS weak signal	BS1 ≈ BS2 ≈ BS3, weaker signal due to distance	3, depending on wall thickness, might not be possible to distinguish between inside/outside	MS{x,y} ∈ triangle if great distances between BS or NLOS, MS{x,y} ∉ triangle else
2	LOS with similar signal strength	BS1 ≈ BS2 ≈ BS3, strong signal		MS{x,y} ∈ triangle
3	NLOS similar strength	BS1 ≈ BS2 ≈ BS3, weaker signal due to NLOS	1, depending on walls might not be possible to distinguish between inside/outside	MS{x,y} ∈ triangle with blocking walls or MS{x,y} ∉ triangle being far outside
4	LOS with different signal strength	BS1 > BS2 ≈ BS3	5, might be distinguished with signal strength (BS 1(4) > BS1(5))	MS{x,y} ∈/∉ triangle, not possible to distinguish between the two outcomes, MS is near to BS however
5	NLOS different signal strength	BS1 > BS2 ≈ BS3	4, might be distinguished with signal strength (BS 1(4) > BS1(5))	MS{x,y} ∈/∉ triangle

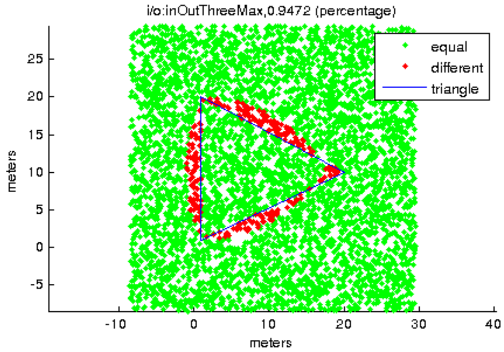
App III. Plots Inside/outside-tests

11: Isosceles

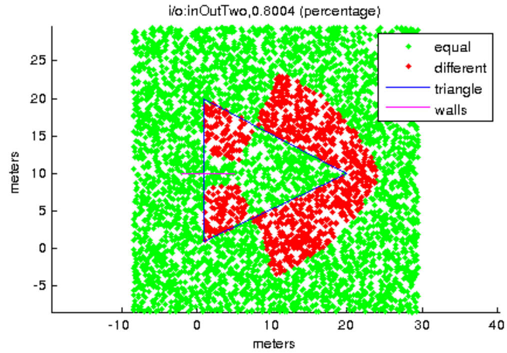


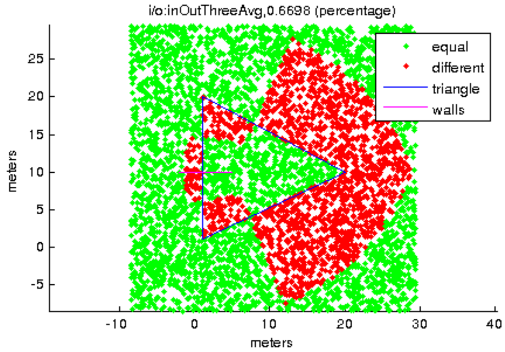


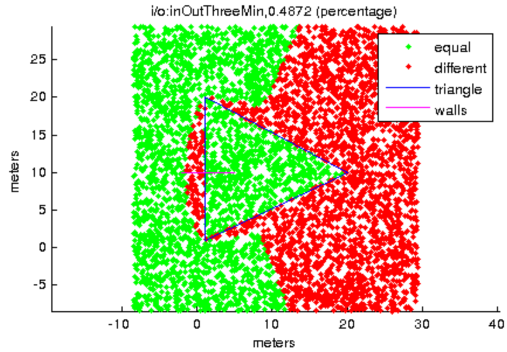


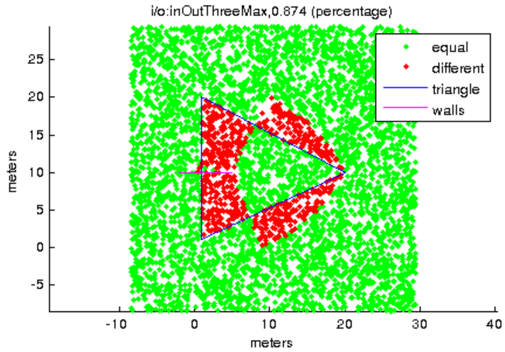


12: Isosceles, one link blocked

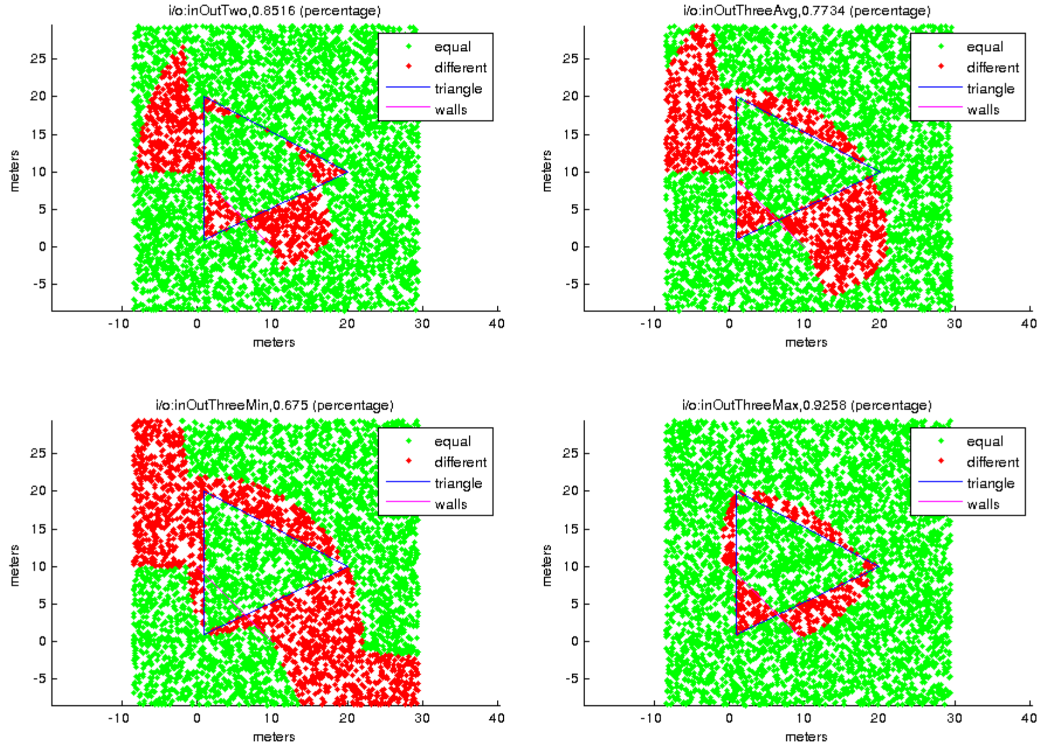




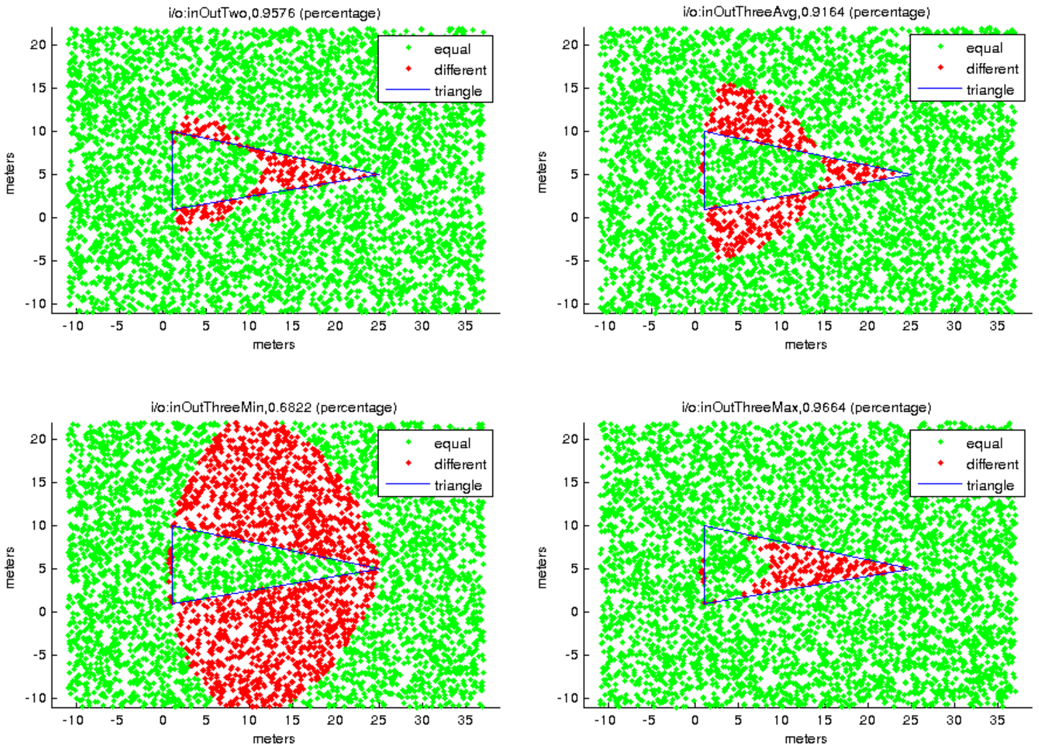




13: Isosceles, two links blocked

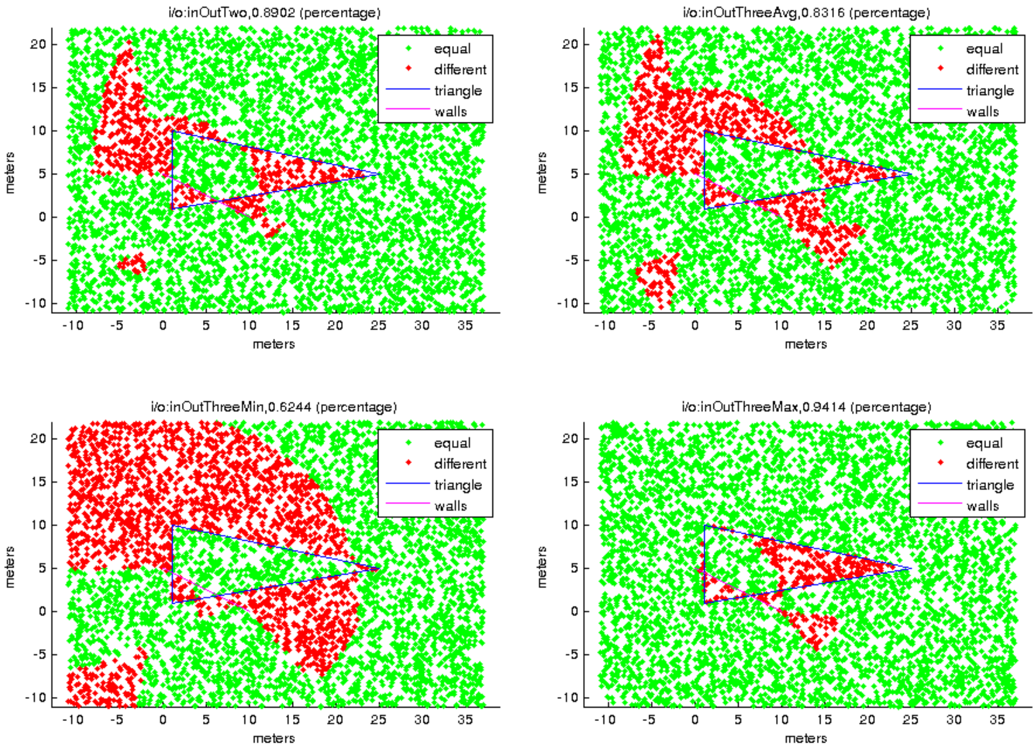


21: Pointy

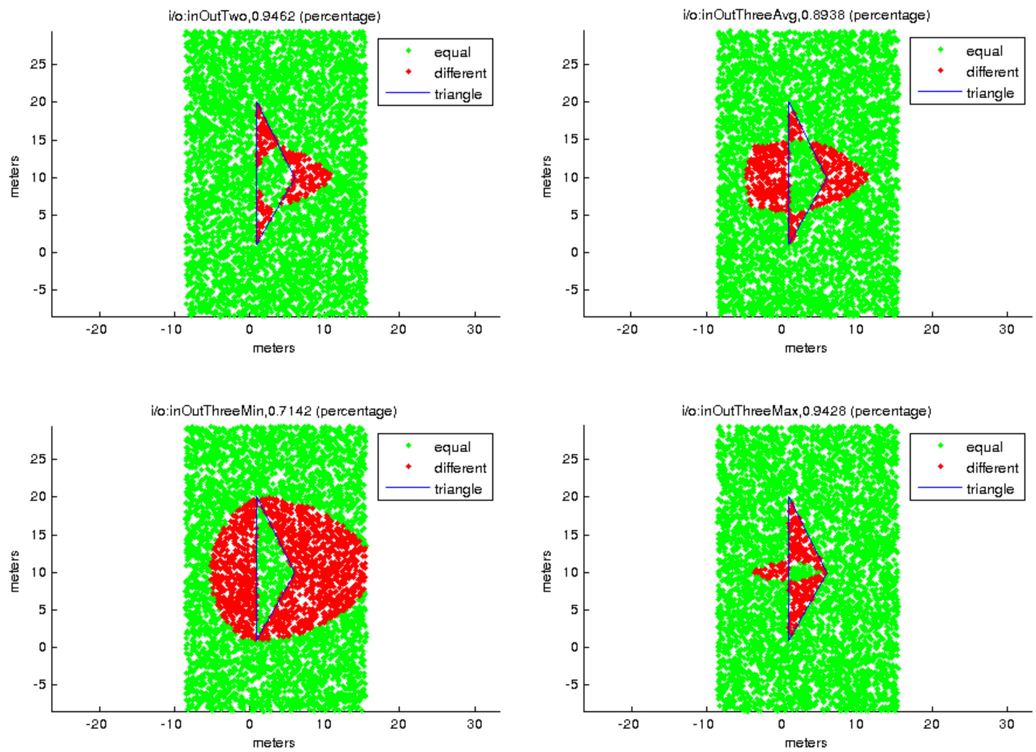


22: Pointy, one link blocked: see Figure 17

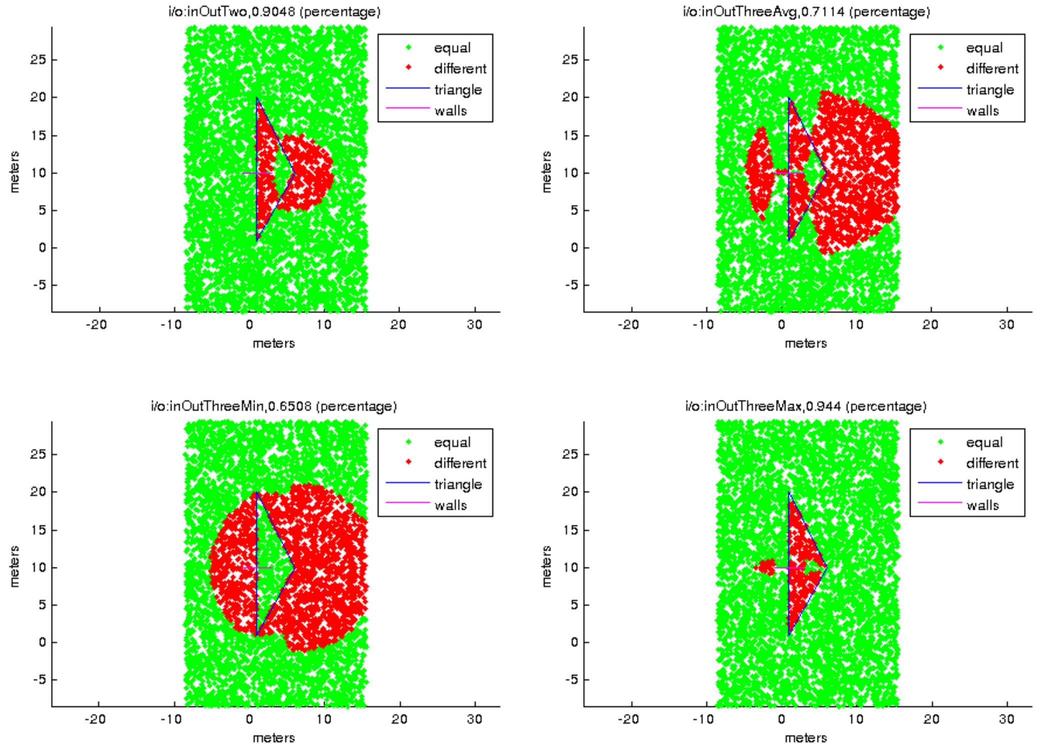
23: Pointy, two links blocked



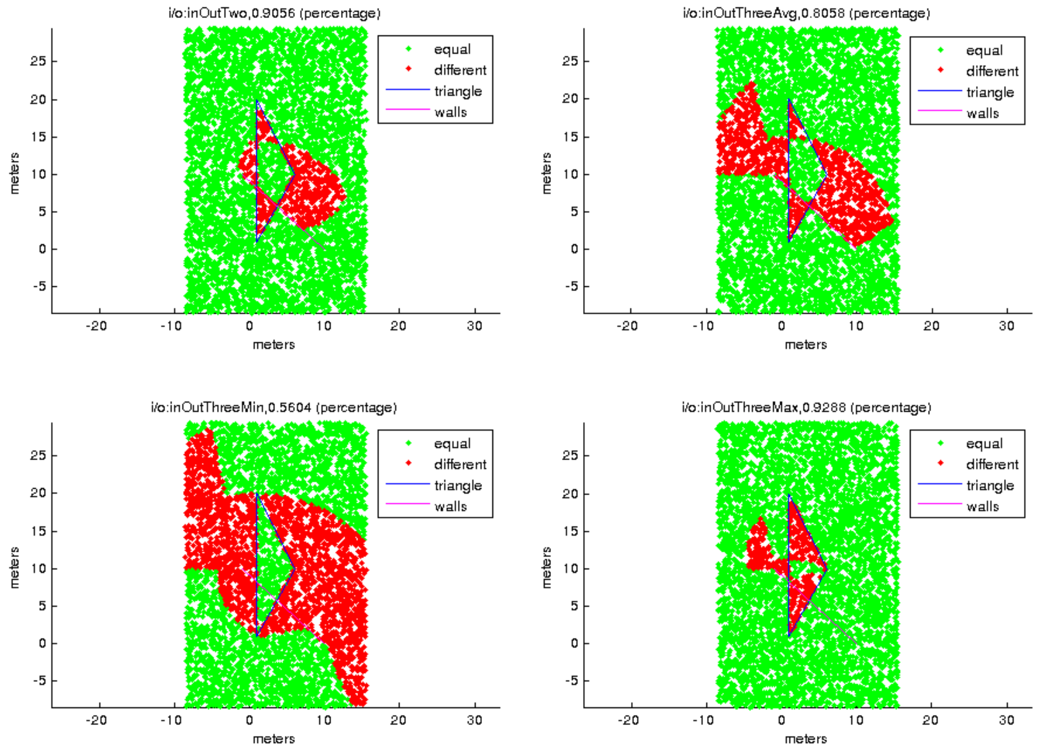
31: Flat



31: Flat, one link blocked



32: Flat, two links blocked



App IV. BS positions

BS-Name	x-pos [m]	y-pos [m]
1	11.2	3.7
2	6.2	3.7
3	8.6	18.2
4	12.0	16.8
5	14.3	17.7
7	13.1	9.7

App V. Evaluation positions

Name	x-pos [m]	y-pos [m]	Time from	Time to	Total N of entries
203	5.2	9.4	08:48	08:52	70
Hall left	6.8	11.6	08:53	08:57	10
202	9.0	9.4	08:58	09:03	62
204	8.6	13.4	09:04	09:08	15
205	10.0	13.8	09:09	09:13	14
Hall right	14.5	11.0	09:14	09:19	20
206	15	13.8	09:20	09:26	33
207	18.4	13.8	09:27	09:31	48

App VI. Inter BS readings

Node pair	Distance [m]	Count	Variance
same node	0	233	145,2
45	2,47	20	10,0
34	3,677	44	15,6
12	5	119	68,2
35	5,722	57	15,5
17	6,294	90	12,0
47	7,185	47	22,7
57	8,089	51	19,9
27	9,144	29	29,2
37	9,618	10	33,1
14	13,124	58	19,5
24	14,327	10	41,8
15	14,339	57	40,8
23	14,697	48	30,8
13	14,731	11	30,0
25	16,174	7	24,0

App VII. RSSI-readings MS to BS

RSSI min [dB]

Location	Time Band	BS 1	BS 2	BS 3	BS 4	BS 5	BS 7
203	TB1	-92	-78	-83	-77	-86	-86
hall left	TB2	-77	-76	-72	-80	-76	-72
202	TB3	-89	-77	-91	-94	-87	-84
204	TB4	-82	-82	-93	-81	-81	-75
205	TB5	-81	-77	-82	-79	-76	-73
hall right	TB6	-79	-75	-85	-68	-70	-68
206	TB7	-80	-76	-81	-78	-75	-73
207	TB8	-73	-87	-94	-90	-85	-88
NA	NA	-88	-90	-94	-90	-84	-81

RSSI max [dB]

Location	Time Band	BS 1	BS 2	BS 3	BS 4	BS 5	BS 7
203	TB1	-69	-55	-58	-65	-62	-64
hall left	TB2	-74	-73	-53	-80	-76	-65
202	TB3	-67	-60	-83	-70	-72	-60
204	TB4	-74	-72	-86	-81	-80	-69
205	TB5	-71	-77	-76	-61	-76	-68
hall right	TB6	-71	-67	-61	-60	-63	-56
206	TB7	-77	-70	-66	-61	-59	-66
207	TB8	-73	-59	-64	-61	-67	-68
NA	NA	-68	-61	-57	-57	-63	-59

RSSI avg [dB]

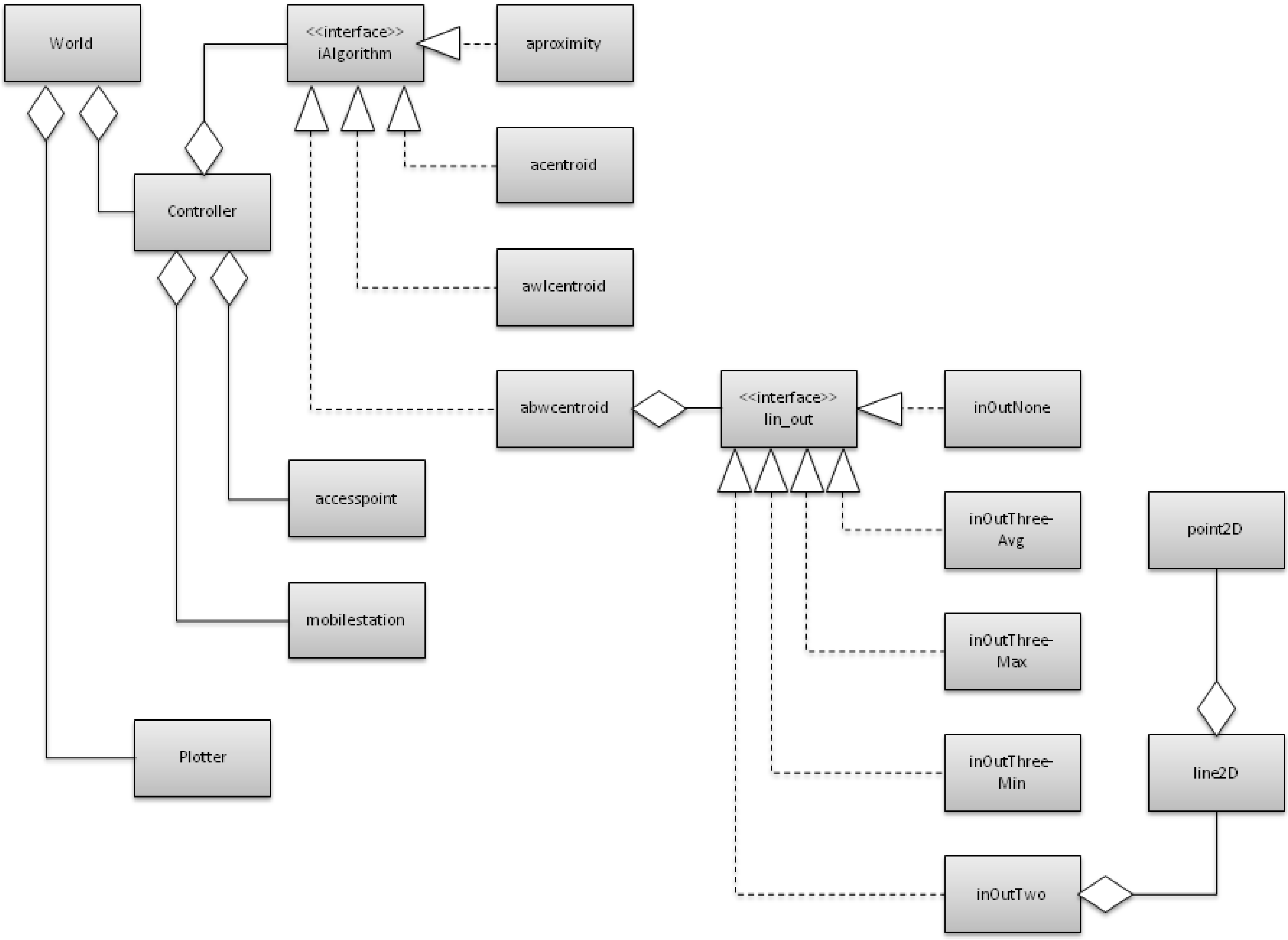
Location	Time Band	BS 1	BS 2	BS 3	BS 4	BS 5	BS 7
203	TB1	-92	-78	-83	-77	-86	-86
hall left	TB2	-77	-76	-72	-80	-76	-72
202	TB3	-89	-77	-91	-94	-87	-84
204	TB4	-82	-82	-93	-81	-81	-75
205	TB5	-81	-77	-82	-79	-76	-73
hall right	TB6	-79	-75	-85	-68	-70	-68
206	TB7	-80	-76	-81	-78	-75	-73
207	TB8	-73	-87	-94	-90	-85	-88
NA	NA	-88	-90	-94	-90	-84	-81

RSSI variance [dB]

Location	Time Band	BS 1	BS 2	BS 3	BS 4	BS 5	BS 7
203	TB1	73,6	55,9	52,9	16,4	61,3	40,9
hall left	TB2	4,5	4,5	180,5	-*	-*	24,5
202	TB3	42,6	27,3	6,1	50,7	22,4	48,8
204	TB4	32,0	28,0	24,5	-*	0,5	4,8
205	TB5	50,0	-*	18,0	63,8	-*	6,3
hall right	TB6	14,9	32,0	160,3	19,0	14,3	21,2
206	TB7	3,0	9,0	112,5	35,4	22,7	6,2
207	TB8	-*	84,8	155,2	92,5	29,7	29,3
NA	NA	51,3	69,9	154,7	104,2	38,7	23,0

*: only one reading

App VIII. UML



Most of the components of the UML will be introduced in the corresponding chapters. Main Components are the central control-unit *controller* acting as the central logical station of the application. Here BS (*accesspoints*) are registered. Furthermore the MS to track (*mobilestation*) is registered too. The controller also keeps track of the *algorithm* in use (by means of an interface accepting multiple algorithms). The most complex algorithm, *abwcentroid*, makes use of an interface (*iin_out*) to integrate inside/outside-tests. *Line2D* and *point2D* are helper-classes for the inside/outside test used in geometrical calculations. The *plotter*-class helps with the creation of the exports (charts and data-files). The *world*-class is the scope or container of the application. It is used for setup-purposes (initialisation of variables) and aggregates all information for simpler access and output. It therefore allows for running different estimations (algorithms) side by side without interference.